

Quiz 1

A Term, 2018

High:	20
Median:	20
Low:	16

Show all work needed to reach your answers.

(5 points each) As $n \rightarrow \infty$, please determine whether each of the following sequences $\{a_n\}$ converges to a limit L or diverges. If it converges, please give the limit, L .

1. $a_n = \frac{2n}{5n-3}$

$$\lim_{n \rightarrow \infty} \frac{2n}{5n-3} = \lim_{n \rightarrow \infty} \frac{2}{5-\cancel{3/n}} = \frac{2}{5}$$

$$L = \frac{2}{5}$$

2. $a_n = 2n^2 \sin(3/n^2)$

$$\lim_{n \rightarrow \infty} 2n^2 \sin(3/n^2) = 2 \lim_{\theta \rightarrow 0^+} \frac{\sin(\theta)}{\frac{1}{3}(3/n^2)} = 6 \lim_{\theta \rightarrow 0^+} \frac{\sin \theta}{\theta}$$

$$L = 6$$

3. $a_n = \left(\frac{n-1}{n+1}\right)^n$

Suppose $L = \lim_{n \rightarrow \infty} \left(\frac{n-1}{n+1}\right)^n$. Then $\ln(L) = \lim_{n \rightarrow \infty} n \ln\left(\frac{n-1}{n+1}\right)$

$$= \lim_{n \rightarrow \infty} \frac{\ln(n-1) - \ln(n+1)}{1/n} = \lim_{n \rightarrow \infty} \frac{\frac{1}{n-1} - \frac{1}{n+1}}{-1/n^2} = - \lim_{n \rightarrow \infty} n^2 \frac{(n+1) - (n-1)}{n^2 - 1}$$

$$= -2 \lim_{n \rightarrow \infty} \frac{n^2}{n^2 - 1} = -2$$

L'Hôpital Rule

$$\text{So... } L = e^{-2} = \frac{1}{e^2}$$

4. $a_n = \frac{1 + (-1)^n}{\cos n}$

odds: $a_{2n-1} = 0 \quad \forall n \in \mathbb{Z}^+$

evens: $a_{2n} = \frac{2}{\cos(2n)}$

$\cos(2n) \neq 0 \quad \forall n \in \mathbb{Z}^+$, but it does approach both 1 and -1 arbitrarily often, so... a_n diverges