MA1033 Theoretical Calculus III   Name Solutions

Quiz 2   A Term, 2018

High: 20
Median: 19
Low: 10

Show all work needed to reach your answers.

(5 points each) As $n \to \infty$, please find the value of the limit L if the sequence converges, or find that the sequence diverges. Also name any test or rule that you use.

1. $a_n = \frac{n}{3^n}$

Notice that $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| \overset{+1}{=} \lim_{n\to\infty} \frac{n+1}{3^{n+1}} \cdot \frac{3^n}{n} = \frac{1}{3}\lim\left(1+\frac{1}{n}\right) \overset{+1}{=} \frac{1}{3} < 1$.

So by the ratio test, (+1)

One can also use l'Hôpital's rule.   (+2) $L = 0$

2. $a_n = (-1)^n \frac{2n^2}{n^2 - 5}$

Ratio test will not help here because it's a rational expression.

Notice that $a_{2n} \to 2$ while $a_{2n-1} \to -2$. (+3)

Hence this sequence ...

(+2) diverges.

3. $a_n = \frac{\ln 2n}{\ln 3n}$

$$L = \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\ln(2n)}{\ln(3n)} \overset{\text{l'Hôpital}}{=} \lim_{n\to\infty} \frac{\frac{2}{2n}}{\frac{3}{3n}} = \lim_{n\to\infty} \frac{n}{n} = 1$$

(+1) l'Hôpital (+1) (+1) (+1)

$L = 1$ (+1)

4. $a_n = \left(\frac{2}{n}\right)^{\frac{2}{n}}$

Let $L = \lim_{n\to\infty} \left(\frac{2}{n}\right)^{2/n}$. Then $\ln(L) \overset{+1}{=} \lim_{n\to\infty} \ln\left(\frac{2}{n}\right)^{2/n}$

$$\overset{+1}{=} \lim_{n\to\infty} \frac{2}{n}\ln\left(\frac{2}{n}\right) \overset{+1}{=} 2\lim_{n\to\infty} \frac{[\ln(2) - \ln(n)]}{n} \overset{+1}{=} 2\lim_{n\to\infty} \frac{[0 - \frac{1}{n}]}{1} = 0$$

l'Hôpital

Since $\ln(L) = 0$ ...

$L = 1$ (+1)