MA1033 Theoretical Calculus III Name Solutions

Quiz 4 A Term, 2018

High: 20
Median: 19
Low: 14

(5 points each) For each series, please state whether it converges or diverges (circle one), which test(s) you use to reach your conclusion, and if possible what limit L the series converges to. Show all work needed to reach your answers. If you need the value of any limit to reach your answer, please compute that value.

1. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

By the integral test, since (+1)

$$\int_2^{\infty} \frac{dx}{x \ln x} \overset{u=\ln x}{=} \int_{\ln(2)}^{\infty} \frac{du}{u} = \lim_{M\to\infty} \left(\ln(u) \right)\Big|_{\ln(2)}^{M} = \lim_{M\to\infty} \left(\ln(M) - \ln(\ln(2)) \right)$$ (+1) (+1) (+1)

diverges, $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ also diverges. (+1)

converges (diverges) $L =$ DNE

2. $\sum_{n=1}^{\infty} \frac{3 - \cos n}{n^2 + 1}$

Notice that $2 \le 3 - \cos n \le 4$. Hence

$\frac{3-\cos n}{n^2+1} \le \frac{4}{n^2+1} \le \frac{4}{n^2}$ (+2), and because the p-series $\sum_{n=1}^{\infty} \frac{4}{n^2}$ (+2)

converges, so does the given series (+1)

(converges) diverges $L =$ Difficult to Determine

3. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2 + n}$

Notice that

$$\frac{\sqrt{n}}{n^2+n} \le \frac{\sqrt{n}}{n^2} = n^{-1.5}$$ (+2)

Because $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ is a convergent p-series (+2)

the given series must also diverge. (+1)

(converges) diverges $L =$ _____

4. Please state the integral test.

For a series $\sum_{k=1}^{\infty} a_k$ (+1) where $a_k \ge 0$, if there is a function $f \ge 0$ such that $f(x) \searrow 0$ as $x \to +\infty$ (+1), and if $f(k) = a_k$ (+1), then the series $\sum_{k=1}^{\infty} a_k$ and the integral $\int_0^{\infty} f(x)\,dx$ converge or diverge (+2) together.