MA1033 Theoretical Calculus III Name Solutions

Quiz 1 A Term, 2019

High: 20
Median: 17
Low: 4

Show all work needed to reach your answers.

(5 points each) As $n \to \infty$, please determine whether each sequence $\{a_n\}$ converges to a limit L or diverges. If it converges, please give the limit, L; otherwise state that $\{a_n\}$ diverges.

1. $a_n = \dfrac{n^2 - n + 7}{2n^3 + n^2}$ $= \dfrac{\cancel{\frac{n^2}{n^3}} - \cancel{\frac{n}{n^3}} + \cancel{\frac{7}{n^3}}}{2 + \frac{1}{n}} \to \dfrac{0}{2}$ (+2) (+1)

0 (+1)

2. $a_n = n\sin(\pi/n) = \pi\left(\dfrac{\sin(\pi/n)}{\pi/n}\right)$ (+1)

Let $u = \frac{\pi}{n}$ (+1)

Then $\lim_{n\to\infty} \dfrac{\pi \sin(\pi/n)}{\pi/n} = \pi \lim_{u\to 0} \dfrac{\sin u}{u}$ (+1) (+2)

π (+1)

3. $a_n = \dfrac{\ln n}{\sqrt{n}}$ Let $n = m^2$ $= \dfrac{\ln m^2}{\sqrt{m^2}} = \dfrac{2\ln m}{m}$

So $\lim_{n\to\infty} \dfrac{\ln n}{\sqrt{n}} = \lim_{m\to\infty} \dfrac{2\ln m}{m} = 2\lim_{m\to\infty} \dfrac{\frac{1}{m}}{1} = 0$ (+1)

(+2) simplify (+2) l'Hôpital

0

4. $a_n = \left(1 - \dfrac{1}{n}\right)^n$

So $\ln a_n = n\ln(1 - 1/n)$, and (+1)

$\lim_{n\to\infty} \ln a_n = \lim_{n\to\infty} \dfrac{\ln(1 - 1/n)}{1/n}$ (+1) $= \lim_{n\to\infty} \dfrac{\frac{1}{1-1/n}\left(\frac{1}{n^2}\right)}{-1/n^2}$ (+1) $= -\lim_{n\to\infty} \dfrac{1}{1-1/n} = -1$ (+1) $\Rightarrow \lim_{n\to\infty} a_n = e^{-1}$ (+1)

l'Hôpital

$\dfrac{1}{e}$