MA1033 Theoretical Calculus III Name Solutions

Quiz 2 A Term, 2019

High: 20
Median: 20
Low: 7

Show all work needed to reach your answers.

(5 points each) As $n \to \infty$, please find the value of the limit L if the sequence converges, or find that the sequence diverges. Also name any test or rule that you use.

1. $a_n = \dfrac{3^n}{n!}$ Notice that $\lim\limits_{n\to\infty} \dfrac{|a_{n+1}|}{|a_n|} = \lim\limits_{n\to\infty} \dfrac{3^{n+1}/(n+1)!}{3^n/n!} = \lim\limits_{n\to\infty} \dfrac{3^{n+1}}{3^n} \cdot \dfrac{n!}{(n+1)!}$ +1 +1

$= \lim\limits_{n\to\infty} \dfrac{3}{n+1} = 0$ +1

By the ratio test, $\lim\limits_{n\to\infty} a_n = 0$ +1 +1

0

2. $a_n = \sqrt{n}\left(\sqrt{n+1} - \sqrt{n}\right) \dfrac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}} = \sqrt{n}\dfrac{1}{\sqrt{n+1}+\sqrt{n}} = \dfrac{1}{\sqrt{1+1/n}+1}$ +1 +1 +1

So $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \dfrac{1}{\sqrt{1+\underbrace{1/n}_{\to 0}}+1} = 1/2$ +1 +1

1/2

3. $a_n = (1+2n)^{2/n}$

Consider $\lim\limits_{n\to\infty} \ln|a_n| = \lim\limits_{n\to\infty} \dfrac{2}{n}\ln(1+2n) = 2\lim\limits_{n\to\infty} \dfrac{\ln(1+2n)}{n} = 2\lim\limits_{n\to\infty} \dfrac{\frac{1}{1+2n}\cdot 2}{1}$ +1 +1 +1

l'Hôpital's Rule

$= 4 \lim\limits_{n\to\infty} \dfrac{1}{1+\underbrace{2n}_{\to\infty}} = 0$ +1

So $\lim\limits_{n\to\infty} a_n = e^0 = 1$ +1

1

4. $a_n = \ln 2n - \ln 3n = \ln\left(\dfrac{2n}{3n}\right) = \ln(2/3)$ +2 +2

Since a_n is constant, $a_n \to \ln(2/3)$ +1

ln(2/3)