MA1033 Theoretical Calculus III **Name** Solutions

Quiz 3 **A Term, 2019**

High : 20
Median : 18
Low : 11

Show all work needed to reach your answers.

(5 points each) For each series, please name the series (harmonic, alternating, geometric) and/or the test needed to determine convergence, explain why it converges or diverges (circle one), and if possible to which limit L it converges.

1. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{n+1} n}{\ln n}$ This is an alternating series, but $a_n = \frac{n}{\ln n}$ +1 $\nearrow +\infty$ as $n \to \infty$ (by l'Hôpital +1 or by noting that n grows faster than $\ln n$). +2 So by the Leibniz alternating series test, this series diverges.

converges **diverges** +1 $L =$ DNE

2. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \left(\frac{e}{\pi}\right)^n$ This is an alternating geometric +1 series, with $a = 1$ and +1 $r = -e/\pi$. Since $-1 < r < 1$ +1, this series converges to $L = \frac{a}{1-r} = \frac{1}{1+e/\pi}$

converges +1 diverges $L = \frac{\pi}{\pi + e}$ +1

3. $\displaystyle\sum_{n=1}^{\infty} (-1)^n [1 - n\sin(1/n)]$ This is an alternating series with $a_n = [1 - n\sin(1/n)]$ +1. Since $\lim_{n\to\infty} n\sin(1/n) = \lim_{\theta\to 0^+} \frac{\sin\theta}{\theta} = 1$, +1 $\lim_{n\to\infty} a_n = 0$. By differentiating, one can show that a_n is decreasing. So by the Leibniz alternating series +2 test, this series converges.

converges +1 diverges L ~~=~~ is messy to compute.

4. $\displaystyle\sum_{n=1}^{\infty} \frac{1 + 2^n}{3^n}$ This is the sum of +1 two convergent geometric series.

$$= \sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n + \sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n = \frac{1/3}{1-1/3} + \frac{2/3}{1-2/3}$$ +1

Here $a = 1/3$ and $r = 1/3$ | Now $a = 2/3$ and $r = 2/3$ +1

$$= \frac{1}{2} + 2$$ +1

converges diverges $L =$ 5/2 +1