MA1033 Theoretical Calculus III Name Solutions

Quiz 4 A Term, 2019

High: 20
Median: 16
Low: 9

(5 points each) For each series, please state whether it converges or diverges (circle one), which test(s) you use to reach your conclusion, and if possible what limit L the series converges to. Show all work needed to reach your answers. If you need the value of any limit to reach your answer, please compute that value.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}$

Since $\frac{1}{n+\sqrt{n}} \geq \frac{1}{2n}$ (+2) $\forall n \in \mathbb{Z}^+$, by the comparison test (+1), this series diverges (+2).

Since $\lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{n+\sqrt{n}}} = \lim_{n\to\infty} \frac{n+\sqrt{n}}{n} = \lim_{n\to\infty} 1+\frac{1}{\sqrt{n}} = 1$ (+2), by the limit comparison test (+1), this series diverges (+2).

converges **diverges**

2. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{4n^2+5}$

By the integral test (+1), one can decide converge for this series by considering $\int_1^{\infty} \frac{x}{4x^2+5}\,dx = \frac{1}{8}\int_9^{\infty} \frac{du}{u} = \frac{1}{8} \lim_{b\to\infty} (\ln b - \ln 9) = +\infty$ (+3). Hence this series diverges (+1).

Since $\frac{n}{4n^2+5} > \frac{n}{4n^2+5n^2} = \frac{1}{9}\cdot\frac{1}{n}$ (+3) $\forall n \in \mathbb{Z}^+$, by the comparison test (+1), this series diverges (+1) ($\sum_{n=1}^{\infty}\frac{1}{n}$ is the harmonic series).

converges **diverges**

3. $\displaystyle\sum_{k=0}^{\infty} \frac{k-1}{k+1}$

Since $\lim_{k\to\infty} \frac{k-1}{k+1} = 1$ (+2), by the k-th term test (+1), this series diverges (+2).

converges **diverges**

4. $\displaystyle\sum_{k=2}^{\infty} \frac{(-1)^k}{k\sqrt{\ln k}}$

This is an alternating series. By the Leibniz alternating series test (+1), since $\frac{1}{k\sqrt{\ln k}} \searrow 0$ (+2), this series converges (+2).

converges diverges