MA1033 Theoretical Calculus III Name Solutions

Quiz 5 A Term, 2019

High: 20
Median: 19
Low: 13

(5 points each) For each power series, please find x_o, the center of the interval of convergence, and R, the radius of convergence. Show all work needed to reach your answers. If you need the value of any limit to reach your answer, please compute that value.

1. $\sum_{n=1}^{\infty} \frac{n^{10}x^n}{10^n}$

$$L = \lim_{n\to\infty} \frac{\frac{(n+1)^{10}}{10^{n+1}}}{\frac{n^{10}}{10^n}} = \lim_{n\to\infty} \frac{10^n}{10^{n+1}}\left(1+\frac{1}{n}\right)^{10} = 1/10$$

+1 +1 +1

$x_o =$ 0 +1 $R =$ 10 +1

2. $\sum_{n=1}^{\infty} \frac{n!}{2^n}(x-5)^n$

$$L = \lim_{n\to\infty} \frac{\frac{(n+1)!}{2^{n+1}}}{\frac{n!}{2^n}} = \lim_{n\to\infty} \frac{\cancel{2^n}}{2^{n+1}} \cdot \frac{(n+1)\cancel{!}}{\cancel{n!}} = \frac{1}{2}\lim_{n\to\infty} n+1 = +\infty$$

+1 +1 +1

$x_o =$ 5 +1 $R =$ 0 +1

3. $\sum_{k=0}^{\infty} \frac{k+1}{k!}(x-a)^k$

$$L = \lim_{k\to\infty} \frac{\frac{k+2}{(k+1)!}}{\frac{k+1}{k!}} = \lim_{k\to\infty} \frac{k+2}{k+1} \cdot \frac{\cancel{k!}}{(k+1)\cancel{!}} = \lim_{k\to\infty} \frac{k+2}{(k+1)^2} = \lim_{k\to\infty} \frac{k+2}{k^2+2k+1} = 0$$

+1 +1 +1

$x_o =$ a +1 $R = +\infty$ +1

4. For the following series (technically not a power series), please find the convergence interval.

$\sum_{n=1}^{\infty} \left(\frac{x^3-1}{7}\right)^n$

By either the ratio test, or because this is a geometric series, it converges provided that $\left|\frac{x^3-1}{7}\right| < 1$ +2 $\iff -7 < x^3-1 < 7 \iff -6 < x^3 < 8$

$\iff -\sqrt[3]{6} < x < 2$ +3

Interval of Convergence: $x \in (-\sqrt[3]{6}, 2)$