MA1033 Theoretical Calculus III Name *Solutions*

Quiz 6 A Term, 2019

High 20
Median 18
Low 5

Show all work needed to reach your answers. For questions #1 and #2, please find a power series representation for each function, then for #1, give the radius of convergence. For questions #3, please compute the indicated limit. For questions #4, please find the first **three** terms of the series expansion.

1. (5 points) $f(x) = \sin(x^2)$ $= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!}(x^2)^{2k+1}$ +3

Because $R=+\infty$ for the sin u Taylor series, the same is true here.

+1 +1

Series: $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{4k+2}$ $R =$ $+\infty$

2. (5 points) $f(x) = \int_0^x \frac{\cos(-t^2) - 1}{t} dt$

$\frac{\cos(-t^2) - 1}{t} = \frac{1}{t}\sum_{k=\cancel{0}\,1}^{\infty} \frac{(-1)^k}{(2k)!}(-t^2)^{2k} = \sum_{k=1}^{\infty} \frac{(-1)^k}{(2k)!}(t)^{4k-1}$ +3

So $\int_0^x \frac{\cos(-t^2)-1}{t} dt = \int_0^x \left(\sum_{k=1}^{\infty} \frac{(-1)^k}{(2k)!} t^{4k-1}\right) dt = \sum_{k=1}^{\infty} \frac{(-1)^k}{(2k)!} \int_0^x t^{4k-1} dt = \sum_{k=1}^{\infty} \frac{(-1)^k}{(2k)!} \frac{t^{4k}}{4k}\Big|_0^x$ +1

Series: $\sum_{k=1}^{\infty} \frac{(-1)^k x^{4k}}{(2k)!(4k)}$ +1

3. (5 points) $\lim_{x\to 0} \frac{1 - \cos x}{x(e^x - 1)}$ $= \lim_{x\to 0} \frac{\cancel{1} - \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^{2k}}{x\left(\sum_{k=\cancel{0}\,1}^{\infty} x^k/k! - \cancel{1}\right)}$ +3

$= \lim_{x\to 0} \frac{-\left(-\frac{1}{2}x^2 + \frac{1}{4!}x^4 - O(x^6)\right)}{x\left(x + \frac{x^2}{2} + O(x^3)\right)} = \lim_{x\to 0} \frac{\frac{1}{2} - \frac{1}{4!}x^2 + O(x^4)}{1 + \frac{x}{2} + O(x^2)}$

+1

+1

Limit: $1/2$

4. (5 points) $\int_0^1 \frac{\sin x}{x} dx$ $= \int_0^1 \frac{\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+\cancel{1}}}{\cancel{x}} dx = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \int_0^1 x^{2k} dx$ +1 +1 +1

$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \frac{x^{2k+1}}{2k+1}\Big|_0^1 = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \frac{1}{(2k+1)} = 1 - \frac{1}{3!\cdot 3} + \frac{1}{5!\cdot 5} + \ldots$

+1 +1

Expansion: $1 - \frac{1}{18} + \frac{1}{600}$