MA1033 Theoretical Calculus III

Name Solutions

Quiz 1

High: 20
Median: 19

A Term, 2015

Show all work needed to reach your answers.

(5 points each) As $n \to \infty$, please determine whether each of the following sequences $\{a_n\}$ converges to a limit L or diverges. If it converges, please give the limit, L.

1. $a_n = \frac{\sin n}{3^n}$

Since $|\sin n| \leq 1$, $|a_n| \leq \frac{1}{3^n} \searrow 0$ as $n \to \infty$

$L = 0$

2. $a_n = n \sin(\pi/n)$

$$\lim_{n\to\infty} n \sin\left(\frac{\pi}{n}\right) = \lim_{n\to\infty} \frac{\sin\left(\frac{\pi}{n}\right)}{1/n}$$

$$= \pi \lim_{n\to\infty} \frac{\sin\left(\frac{\pi}{n}\right)}{\pi/n} = \pi \underbrace{\lim_{x\to 0} \frac{\sin x}{x}}_{1}$$

Let $x = \frac{\pi}{n}$

$L = \pi$

3. $a_n = \frac{2^n + 1}{e^n} = \left(\frac{2}{e}\right)^n + \left(\frac{1}{e}\right)^n$

Since both $\frac{2}{e} < 1$ and $\frac{1}{e} < 1$, both $\left(\frac{2}{e}\right)^n \searrow 0$ and $\left(\frac{1}{e}\right)^n \searrow 0$

$L = 0$

4. $a_n = \left(1 + \frac{2}{n}\right)^n = \left(\left(1 + \frac{1}{n/2}\right)^{n/2}\right)^2$

So $$\lim_{n\to\infty} a_n = \left(\lim_{n\to\infty} \left(1 + \frac{1}{n/2}\right)^{n/2}\right)^2$$

$$= \left(\underbrace{\lim_{m\to\infty} \left(1 + \frac{1}{m}\right)^m}_{e}\right)^2$$

$L = e^2$