MA1034 Theoretical Calculus IV **Name** Solutions

Quiz 2 **B Term, 2017**

High : 25
Median : 21
Low : 18

Show all work needed to reach your answers.

#5
§9.6
p.415

1. (8 points) Please find all unit vectors that are tangent to the curve $y = x^3 - x$ at the point $(1, 0)$. Hint: Consider $\boldsymbol{x}(t) = \langle t, t^3 - t\rangle$. $\Rightarrow$ $\vec{V}(t) = \vec{X}'(t) = \langle 1, 3t^2 - 1\rangle$ +2

The curve goes through $(1,0)$ when $t = 1$ +1, so $\vec{V}(1) = \langle 1, 2\rangle$.
Thus $\hat{T}(1) = \frac{\vec{V}(1)}{\|\vec{V}(1)\|}$ +3 $= \langle \sqrt{5}/5, \frac{2\sqrt{5}}{5}\rangle$ is a unit vector tangent to this curve. The other unit vector tangent to this curve is its negative: +2 $\langle -\sqrt{5}/5, -\frac{2\sqrt{5}}{5}\rangle$.

#10
§2
p.36

2. (7 points) Consider the helix traced out by the vector function $\boldsymbol{x}(t) = \langle 2\cos t, 2\sin t, 3t\rangle$. Please find $s(t)$.

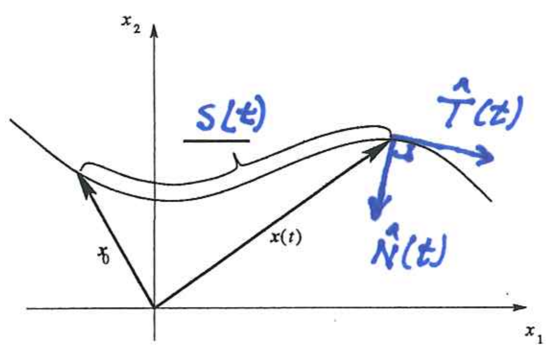
By definition: $s(t) :=$ +3 $\int_0^t \|\vec{X}'(\tau)\|\,d\tau = \int_0^t \sqrt{(-2\sin\tau)^2 + (2\cos\tau)^2 + 3^2}\,d\tau$ +2

$= \int_0^t \sqrt{4\underbrace{(\sin^2\tau + \cos^2\tau)}_{1} + 9}\,d\tau$ +1 $= \sqrt{13}\,t$

$s(t)$: $\sqrt{13}\,t$ +1

3. (10 points) For the vector function $\boldsymbol{x}(t)$, please complete the following table and diagram, drawing and labeling $\boldsymbol{T}(t)$ and the unit normal vector:

1 point each, max 10



Symbol	Defined as (in symbols)	Name
$\boldsymbol{T}(t)$	$\frac{\vec{V}(t)}{\lvert\vec{V}(t)\rvert} \equiv \frac{\vec{X}'(t)}{\lvert\vec{X}'(t)\rvert}$	unit tangent vector
$\hat{N}(t)$	$\vec{T}'(t)/\|\vec{T}'(t)\|$	unit normal vector
$s'(t)$	$\lvert\vec{X}'(t)\rvert = \lvert\vec{V}(t)\rvert$	speed
$K(s)$	$\lvert\frac{d\boldsymbol{T}}{ds}\rvert$	curvature.