MA1034   Theoretical Calculus IV   Name ___Solutions___

Quiz 5   B Term, 2017

High: 25
Median: 20
Low: 14
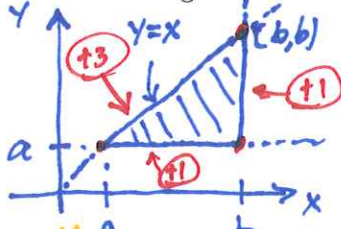
Show all work needed to reach your answers.

1. (7 points) If $f(x,y)$ is continuous and $[a,b]$ is an interval, then *Dirichlet's formula* states that

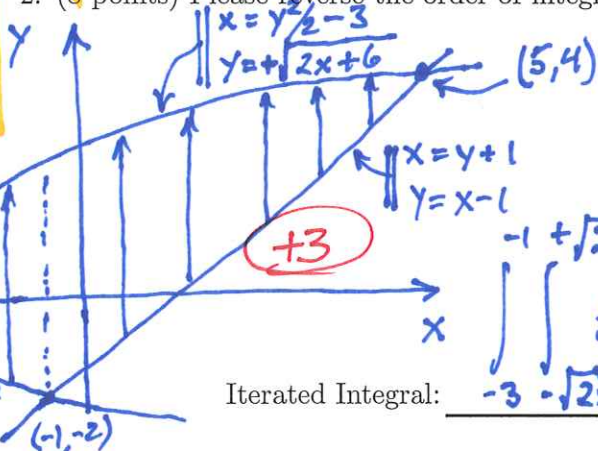$$\int_a^b \int_a^x f(x,y)\,dy\,dx = \int_a^b \int_y^b f(x,y)\,dx\,dy$$

#3
§11.3
p. 495

What region do these iterated integrals represent? That is, what region is the corresponding double integral over?

It's the triangle bounded below by $y=a$, to the right by $x=b$, and to the left/top by $y=X$.

#2a
§11.3
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2. (8 points) Please reverse the order of integration: $\int_{-2}^{4} \int_{y^2/2-3}^{y+1} f(x,y)\,dx\,dy$

$$\int_{-3}^{-1}\int_{-\sqrt{2x+6}}^{+\sqrt{2x+6}} f(x,y)\,dy\,dx \;+\; \int_{-1}^{5}\int_{x-1}^{+\sqrt{2x+6}} f(x,y)\,dy\,dx$$

Iterated Integral: _____

3. (10 points) Please set up and evaluate a double integral representing the volume under the surface $z = 1 + x^2 + y^2$ and above the semicircular disk in the x,y-plane between the x-axis $(y=0)$ and the semicircle $y = +\sqrt{4-x^2}$.

$$V = \int_0^\pi \int_0^2 (1+r^2)\,r\,dr\,d\theta = \left(\int_0^\pi d\theta\right)\left(\int_0^2 r + r^3\,dr\right) = \pi\left(\frac{r^2}{2} + \frac{r^4}{4}\right)\Big|_0^2$$

$$= \pi\left(2 + 4\right) = 6\pi$$