MA144X Theoretical Calculus ~~III~~ IV   Name Solutions

Quiz 1   B Term, 2013

Show all work needed to reach your answers.

High: 25
Median: 20
Low: 8

1. (6 points) If $\mathbf{u} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ and $\mathbf{v} = 4\mathbf{i} - \mathbf{j} + 2\mathbf{k}$, please compute $\mathbf{u} \times \mathbf{v}$.

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 4 & -1 & 2 \end{vmatrix} = \langle 2+3, -(4-12), -2-4 \rangle$$

sign errors: -1, -2 or -3

$|\vec{u} \times \vec{v}| = 19$: 2

$\langle 5, 8, -6 \rangle$

2. (6 points) When is $|\mathbf{u} \cdot \mathbf{v}| = \|\mathbf{u}\| \, \|\mathbf{v}\|$? Please explain. This holds for $\vec{u} = \vec{0}$ or $\vec{v} = \vec{0}$. Otherwise $|\vec{u} \cdot \vec{v}| = \|\vec{u}\| \|\vec{v}\| |\cos\theta|$. So $|\vec{u} \cdot \vec{v}| = \|\vec{u}\| \|\vec{v}\|$ implies that $|\cos\theta| = 1 \Leftrightarrow \cos\theta = \pm 1$. So $\theta = 0$ or $\theta = \pi \Leftrightarrow$ $\vec{u}$ and $\vec{v}$ are co-linear, either parallel or antiparallel. $\cos\theta = 1$: -2

3. (8 Points) Do the curves $\gamma_1(t) = (2t - 1, t^2)$ and $\gamma_2(t) = (4t, 1)$ intersect? If so, at which point(s)?

For these two curves to interesect, $\exists\, t_1, t_2 \in \mathbb{R}$ s.t. $\gamma_1(t_1) = \gamma_2(t_2) \Leftrightarrow (2t_1 - 1, t_1^2) = (4t_2, 1)$. So

$$\begin{cases} 2t_1 - 1 = 4t_2 \\ t_1^2 = 1 \end{cases} \Leftrightarrow \begin{array}{l} t_1 = \pm 1 \\ t_2 = \frac{t_1}{2} - \frac{1}{4} \end{array}.$$

Thus there are two intersection points: $(1, 1)$ and $(-3, 1)$

just found t_1, t_2 : -2

found only 1 point : -4