

Quiz 3

B Term, 2013

Show all work needed to reach your answers.

1. (17 points) Please compute each limit or explain why the limit does not exist.

5 (a) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y^2}{x^2 + y^2} = \lim_{r \rightarrow 0} \frac{r^2(\cos^2 \theta - \sin^2 \theta)}{r^2} = \cos^2 \theta - \sin^2 \theta$

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$r^2 = x^2 + y^2$$

Since this solution

depends on θ , the angle of approach, the limit...

High: 25
Median: 20
Low: 1

6 (b) $\lim_{\substack{(x,y) \rightarrow (1,3) \\ y \neq 3x}} \frac{27x^3 - y^3}{9x^2 - y^2} = \lim_{(x,y) \rightarrow (1,3)} \frac{(3x-y)(9x^2+3xy+y^2)}{(3x-y)(3x+y)} = \frac{9+9+9}{3+3}$

Factor: +2 Limit: DNE +1
Cancel: +1 Factor: +1

6 (c) $\lim_{(x,y) \rightarrow (0,0)} \frac{\tan(5x^2 + y^2)}{x^2 + (y^2/5)} = \lim_{u \rightarrow 0} \frac{\tan u}{u/5} = 5 \cdot \lim_{u \rightarrow 0} \frac{\sin u}{u} \cdot \lim_{u \rightarrow 0} \frac{1}{\cos u} = 5 \cdot 1 \cdot 1$

let $u = 5x^2 + y^2$

Limit: 9/2 +1

Limit: 5 +1

2. (8 points)

- 4 (a) Can the expression in 1(a) be defined to be continuous at
- $(0,0)$
- ? Please explain why or why not.

No, since the limit DNE, the expression can not be defined continuously.

- 4 (b) Can the expression in 1(b) be defined to be continuous at
- $(1,3)$
- ? Please explain why or why not.

Yes, one can extend the definition continuously in several ways, for example,

$$\begin{cases} \frac{27x^3 - y^3}{9x^2 - y^2} & y \neq 3x \\ \frac{9x^2 + 3xy + y^2}{3x + y} & y = 3x \end{cases} = \frac{9x^2 + 3xy + y^2}{3x + y} \quad \forall xy$$