MA144X Theoretical Calculus IV Name Solutions

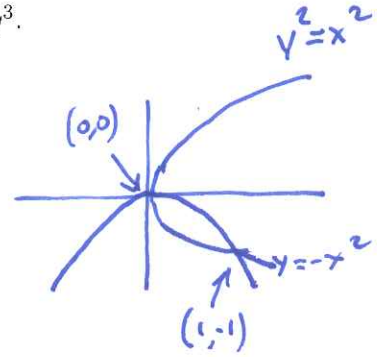
Quiz 4 B Term, 2013

Show all work needed to reach your answers.

1. (10 points) Please find the critical points of $f(x,y) = x^3 + 3xy - y^3$.

$\vec{\nabla} f(x,y) = \langle 3x^2 + 3y, 3x - 3y^2 \rangle = \vec{0}$ (+1) (+1) (+2)

$\begin{cases} x^2 + y = 0 \\ x - y^2 = 0 \end{cases} \Rightarrow \begin{cases} y = -x^2 \\ y^2 = x \end{cases}$ (+1) (+1) (+1)



Critical Point: (0,0) (+1)

Critical Point: (1,-1) (+1)

Critical Point: _____

Critical Point: _____

2. (10 points) Please find an equation of the plane tangent to the surface $z = f(x,y) = x^3 + 3xy - y^3$ at $(1,1,3)$.

$\frac{\partial f}{\partial x}(1,1) = 3x^2 + 3y\Big|_{(1,1)} = 6$ (+2)

$\frac{\partial f}{\partial y}(1,1) = 3x - 3y^2\Big|_{(1,1)} = 0$ (+2)

$z = z_0 + \frac{\partial f}{\partial x}(x_0,y_0)(x - x_0) + \frac{\partial f}{\partial y}(x_0,y_0)(y - y_0)$ (+4)

$= 3 + 6(x-1) + 0(y-1)$ (+1)

$= 3 + 6x - 6 = 6x - 3$

Equation: $z = 6x - 3 \iff 6x - z = 3$ (+1)

3. (5 points) If $z = f(x,y)$, what limit must be zero if f is to be *differentiable* and have a *unique tangent plane* at (x_0, y_0)?

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$$\lim_{(x,y)\to(x_0,y_0)} \frac{f(x,y) - \left[f(x_0,y_0) + \frac{\partial f}{\partial x}(x_0,y_0)(x - x_0) + \frac{\partial f}{\partial y}(x_0,y_0)(y - y_0)\right]}{d\left((x,y),(x_0,y_0)\right)} = 0$$