

Quiz 1

B Term, 2015

High: 25
Median: 19
Low: 11

Show all work needed to reach your answers.

1. (8 points) Suppose that $u(t) = \langle \cos t, \sin t \rangle$, and $v(t) = \langle \cos 2t, \sin 2t \rangle$. Please show that the curve C for u is the same as that for v .

Let $\tau = 2t$. Then $\vec{u}(\tau) = \vec{u}(2t) = \vec{v}(t)$. This guarantees that C is the same for both vector functions. Also notice that since $\cos^2 \theta + \sin^2 \theta = 1 \quad \forall \theta$, both curves lie on the circle $x^2 + y^2 = 1$ (in fact, both are the entire circle).

2. (8 points) Please find the equation in rectangular coordinates that represents the curve C for the parameterization $\gamma(t) = (\cos t, \cos 2t)$. What is the name of this curve? One or more of these trig identities below might be helpful.

$$1 - \sin^2 \theta = \cos^2 \theta$$

$$\cos 2\theta = 2\cos^2 \theta - 1$$

$$\sin 2\theta = 2\sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\gamma(t) = (\cos t, \cos 2t) \Rightarrow \begin{cases} x(t) = \cos t \\ y(t) = \cos 2t \end{cases}$$

$$\cos 2t = 2(\cos t)^2 - 1 \Rightarrow y(t) = 2(x(t))^2 - 1$$

$$\text{Thus } y = 2x^2 - 1 \text{ for } |x| \leq 1,$$

and this is a parabola (or a segment of a parabola).

3. (9 Points) If $u \times i = 0$ and $u \times k = 0$, what can one say about u ? Please explain/justify your answer.

$\vec{u} \times \hat{i} = 0$ and $\vec{u} \times \hat{k} = 0$ means that $\vec{u} \parallel \hat{i}$ and $\vec{u} \parallel \hat{k}$ which is impossible unless $\vec{u} = \vec{0}$.

OR

$$\vec{u} \times \hat{i} = \vec{0} \Leftrightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & u_2 & u_3 \\ 1 & 0 & 0 \end{vmatrix} = \langle 0, u_3, -u_2 \rangle = \langle 0, 0, 0 \rangle, \text{ and}$$

$$\vec{u} \times \hat{k} = \vec{0} \Leftrightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & u_2 & u_3 \\ 0 & 0 & 1 \end{vmatrix} = \langle u_2, -u_1, 0 \rangle = \langle 0, 0, 0 \rangle$$

which means that $u_1 = u_2 = u_3 = 0 \Rightarrow \vec{u} = \vec{0}$.