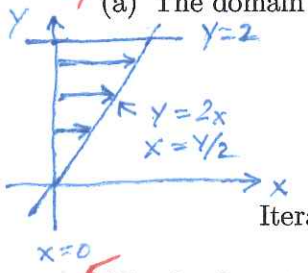
MA1034 Theoretical Calculus IV **Name** Solutions

Quiz 5 **B Term, 2015**

Show all work needed to reach your answers.

1. (10 points) Please set up the iterated integral $\iint f(x,y)\,dx\,dy$ for each $\int\int_D f(x,y)dA$

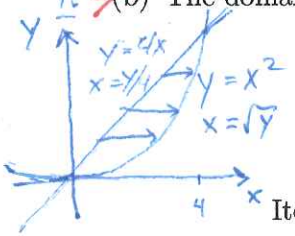
5/ (a) The domain D is the region between the curves $y = 2x$, $x = 0$ and $y = 2$.



High: 24
Median: 21
Low: 7

Iterated Integral: $\int_0^2 \int_{0}^{Y/2} f(x,y)\,dx\,dy$ (+1) (+2) (+1) (+1)

5/ (b) The domain D is the region bounded by the curves $y = x^2$ and $y = 4x$.



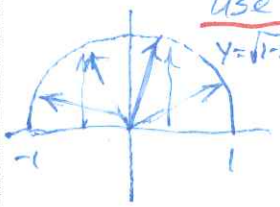
Iterated Integral: $\int_0^{16} \int_{Y/4}^{\sqrt{Y}} f(x,y)\,dx\,dy$ (+1) (+2) (+2)

2. (10 points) Please evaluate

10

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} (x^2+y^2)^{3/2}\,dy\,dx$$

(+1) Use Polar Coordinates:



$\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} (x^2+y^2)^{3/2}\,dy\,dx = \int_{0}^{\pi}\int_{0}^{1} (r^2)^{3/2}\, r\,dr\,d\theta = \int_0^{\pi} d\theta \int_0^1 r^4\,dr = \pi\cdot\frac{1}{5}$ (+1) (+1) (+1) (+1) (+1) (+1) (+1)

$\pi/5$

5 3. (5 points) If $R = [0,1]\times[0,1]$ is a rectangle, and if f is defined on R by

$$f(x,y) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{otherwise} \end{cases}$$

show that f is not Riemann integrable on R.

For this function, $M_{ij}(f,P) = 1$ $\forall i,j$ (+1), no matter what the partition is, because any subrectangle in the partition will contain points whose x coordinate is rational (+1). Similarly $m_{ij}(f,P) = 0$ $\forall i,j$, $\forall P$. So $\iint_R f(x,y)\,dA = 1$ while $\iint_R f(x,y)\,dA = 0$. Since these two are distinct, $\iint_R f(x,y)\,dA$ DNE. (+1) (+1)