MA144X Theoretical Calculus IV **Name:** _____

Final **B Term, 2013**

Show all work needed to reach your answers.

1. (10 points) For $g(x, y, z) = 3xy^2 \cos yz + x/y$, please compute $\partial g/\partial y$

$\partial g/\partial y =$ _____

2. (10 points) For the surface $z = f(x, y) = x^2 - 3xy + 7x - 3y^2 + 8$, please find the critical point and decide if it is a maximum, minimum or a saddle point.

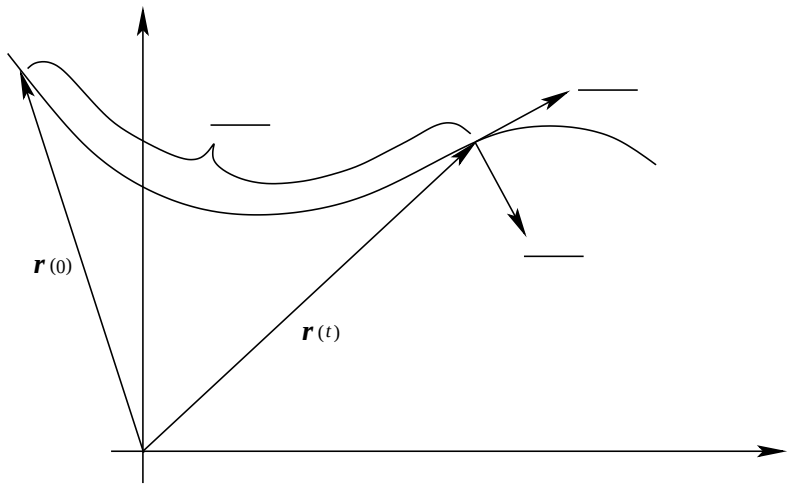
Critical Point: _____

Type: _____

3. (12 points) Please find an equation of the plane passing through the points $(1, 2, 0)$, $(0, 4, 1)$ and $(8, 0, 1)$.

Plane: _____

4. (18 points) Please complete the following table and diagram:



Symbol	Defined as (in symbols)	Name
$\mathbf{T}(t)$	_____	_____
$\mathbf{N}(t)$	$\dfrac{d\mathbf{T}/ds}{\lvert d\mathbf{T}/ds\rvert}$	_____
$s'(t)$	_____	_____
_____	_____	curvature
$\mathbf{v}(t)$	_____	_____

5. (15 points) Please compute $\iint_D \frac{\sqrt{1+x^2}}{x}\, dA$ over the region D between the curves $y = x^2$, $y = 0$ and $x = 2$.

6. (15 points) Consider a hemispherical dome with radius R sitting on top of the x, y-plane: $x^2+y^2+z^2 = R^2,\ z \geq 0$. Suppose that the dome is filled with a gas whose density decreases linearly with height (so $\delta(z) = \delta_0(1 - z/R)$). Please find the mass of this dome.

Mass: _____

7. (10 points) If a function $f : [a, b] \to \mathbb{R}$ is *continuous* at a point $x_0 \in [a, b]$, what is the δ-ϵ definition of *continuous*? Hint: Start with "Given $\epsilon > 0$, . . ."

8. (10 points) Suppose $f(x, y) = 0$ is a smooth curve in $\mathbb{R}^2$ (the x, y-plane), so that f is a differentiable function. Please explain why $\nabla f(x, y)$ is perpendicular to the curve at (x, y).