**MA1034 Theoretical Calculus IV** **Name:** ______________________

**Final** **B Term, 2018**

Show all work needed to reach your answers.

1. (10 points) For $\boldsymbol{u} = \langle 1, 9, -1 \rangle$ and $\boldsymbol{v} = \langle -3, 1, \alpha \rangle$, please find the value of $\alpha$ which make these two vectors orthogonal.

$\alpha =$ ____________

2. (15 points) For the surface $xyz = 6$, please find an equation for the tangent plane at the point $P_0(1, -2, -3)$.

Equation: ______________________

3. (15 points) Please compute $\iiint\limits_{\Omega} z \, dV$ where $\Omega$ is hemisphere under $x^2 + y^2 + z^2 = 4$ and above $z = 0$.

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4. (10 points) For $f(x,y) = x^3 + 5xy^2 - y^3$, please compute the directional derivative $D_{\boldsymbol{u}} f(2,1)$ in the direction of $\langle 1, 1 \rangle$.

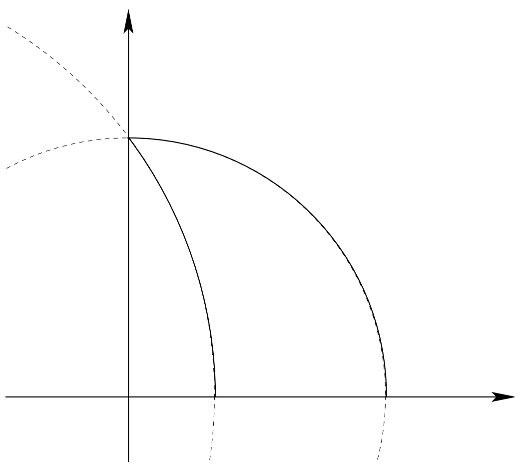
5. (15 points) If the partial derivative exists, please compute $f_y(0,0)$ for

$$f(x,y) = \begin{cases} \dfrac{x^3 - y^3}{x^2 + y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

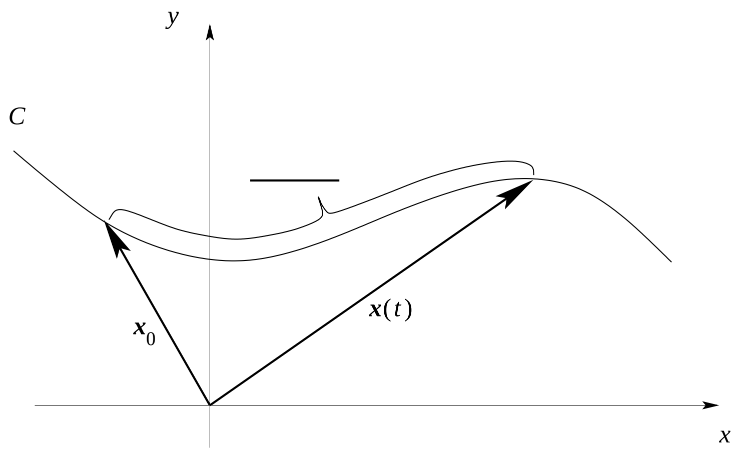
6. (15 points) Set up but **do not evaluate** an iterated integral equal to

$$\iint_D f(x,y)\ dA$$

where $D$ is the domain in the first quadrant between the circles $(x+4)^2+y^2=25$ and $x^2+y^2=9$.



7. (20 points) Suppose $\boldsymbol{x}(t)$ is a differentiable vector function which traces out the curve $C$, and suppose $\boldsymbol{x}'(t) \neq 0$ for any $t$:




(a) Please correctly fill in the blank on the diagram above, and also correctly locate and draw the unit tangent vector $\boldsymbol{T}(t)$ and the unit normal vector $\boldsymbol{N}(t)$.

(b) What are the definitions of $\boldsymbol{T}(t)$ and $\boldsymbol{N}(t)$.

$\boldsymbol{T}(t) =$ ______________________ $\boldsymbol{N}(t) =$ ______________________

(c) Use these definitions to explain why $\boldsymbol{T}(t) \perp \boldsymbol{N}(t)$, that is, prove that $\boldsymbol{T}(t) \perp \boldsymbol{N}(t)$.