MA1034 Theoretical Calculus IV Name Solutions

Quiz 4 B Term, 2017

High : 25
Median : 25
Low : 8

Show all work needed to reach your answers.

#2
§10.6
p.470

1. (12 points) If $z = f(x, y)$ is differentiable, $x = t\sin(ts)$, and $y = s\cos(ts)$, find $\partial z/\partial s$.

$$\frac{\partial z}{\partial s} \overset{+4}{=} \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} \quad \| \text{Chain Rule}$$

$$= f_x(x,y)\left(t^2\cos(ts)\right) + f_y(x,y)\left(\cos(ts) - st\sin(ts)\right) \quad +2 \quad +4$$

+2

$\partial z/\partial s =$ $t^2\cos(ts)\, f_x(x,y) + (\cos(ts) - ts\sin(ts))$

2. (8 points) For $f(x, y) = \sin(xy)$, $(x_0, y_0) = (1, \pi/3)$ and $\boldsymbol{v} = \langle 1, -2\rangle$, please find $D_{\boldsymbol{v}} f(x_0, y_0)$.

$$\vec{\nabla} f(x,y) = \langle y\cos(xy),\ x\cos(xy)\rangle \quad +3$$

+1

$$\vec{\nabla} f(1, \pi/3) = \langle \tfrac{\pi}{3}\cos(\tfrac{\pi}{3}),\ 1\cos(\tfrac{\pi}{3})\rangle = \tfrac{1}{2}\langle \pi/3, 1\rangle$$

$$\hat{v} = \frac{\langle 1,-2\rangle}{\sqrt{1+4}} = \frac{\sqrt{5}}{5}\langle 1,-2\rangle \quad +1$$

+3

Hence $D_{\hat{u}} f(x_0,y_0) = \frac{1}{2}\langle \pi/3, 1\rangle \cdot \frac{\sqrt{5}}{5}\langle 1,-2\rangle = \frac{\sqrt{5}}{10}\left(\frac{\pi}{3} - 2\right)$

$D_{\boldsymbol{v}} f(x_0, y_0) =$ $\frac{\sqrt{5}}{30}(\pi - 6)$

3. (5 points) If $z = f(x, y)$, what limit must be zero if $f$ is to be *differentiable* and have *a unique nonvertical tangent plane* at $(x_0, y_0)$?

+5

$$0 = \lim_{(x,y)\to(x_0,y_0)} \frac{f(x,y) - \left[f(x_0,y_0) + \frac{\partial f}{\partial x}(x_0,y_0)(x-x_0) + \frac{\partial f}{\partial y}(x_0,y_0)(y-y_0)\right]}{d((x,y),(x_0,y_0))}$$