MA1034 Theoretical Calculus IV Name Solutions

Quiz 4 B Term, 2018

Show all work needed to reach your answers.

#2a §10.4 p.461

1. (10 points) Please find an equations of the tangent plane and the normal line to the surface $z = 9 - x^2 - y^2$ at the point $P_0(1, -2, 4)$. Let $F(x,y,z) = z - 9 + x^2 + y^2 = 0$ be the surface (+1)

$\vec{N} = \vec{\nabla} F(1,-2,4) \overset{+2}{=} \langle 2x, 2y, 1\rangle\Big|_{(1,-2,4)} \overset{+1}{=} \langle 2, -4, 1\rangle$

Tangent Plane: $2(x-1) - 4(y+2) + 1(z-4) = 0$ (+3)

High: 25
Median: 24
Low: 14

Tangent Plane Equation: $2x - 4y + z = 14$

Normal Line Equation: $\vec{x}(t) = \langle 1, -2, 4\rangle + \langle 2, -4, 1\rangle t$ (+3) $\begin{cases} x = 2t+1 \\ y = -4t-2 \\ z = t+4 \end{cases}$

2. (10 points) If $z = f(x,y)$ is differentiable, $x = se^{t^2}$, and $y = te^{ts}$, please find $\partial z/\partial t$.

$\frac{\partial x}{\partial t} = 2ts\,e^{t^2}$ (+2) $\frac{\partial y}{\partial t} = e^{ts} + st\,e^{ts} = (1+st)e^{ts}$ (+2)

$\partial z/\partial t = \overset{+3}{} \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} \overset{+1}{=} \frac{\partial z}{\partial x}\left(2ts e^{t^2}\right) + \frac{\partial z}{\partial y}\left((1+st)e^{ts}\right)$

#1a §10.6 p.464

3. (5 points) For $f(x,y) = x + \sin y^2$ and $\boldsymbol{v} = \langle 1, -2\rangle$, please find $D_{\boldsymbol{v}} f(1,0)$.

$\hat{u} = \frac{\vec{v}}{|\vec{v}|} \overset{+1}{=} \frac{\langle 1, -2\rangle}{\sqrt{5}}$ $\quad \vec{\nabla} f(x,y) \overset{+2}{=} \langle 1, 2y\cos y^2\rangle$

$\Rightarrow \vec{\nabla} f(1,0) \overset{+1}{=} \langle 1, 0\rangle$

$D_{\vec{v}} f(1,0) = \vec{\nabla} f(1,0) \cdot \hat{u} \overset{+1}{=} \langle 1, 0\rangle \cdot \frac{\langle 1,-2\rangle}{\sqrt{5}} \overset{+2}{=} \sqrt{5}/5$

$D_{\boldsymbol{v}} f(1, 0) = \sqrt{5}/5$