MA1034 Theoretical Calculus IV **Name:** ____________________

Final **B Term, 2015**

Show all work needed to reach your answers.

1. (10 points) Consider the vectors $\mathbf{v_1} = \langle 3, -1, -5\rangle$, $\mathbf{v_2} = \langle 1, \alpha, -2\rangle$ and $\mathbf{v_3} = \langle -6, \beta, 10\rangle$. For what value of α is $\mathbf{v_1} \perp \mathbf{v_2}$? For what value of β is $\mathbf{v_1} || \mathbf{v_3}$?

$\alpha =$ __________ $\beta =$ __________

2. (10 points) For the surface $z = f(x, y) = x^2 - 3xy + 7x - 3y^2 + 8$, please find an equation for the tangent plane at $(1, -1, 16)$.

Tangent Plane: ____________________

3. (10 points) For the vector function $\mathbf{x}(t) = \langle t + 1, t^2 - t + 2, e^{2t}\rangle$, please find the speed $s'(t) = |\mathbf{v}(t)|$ and the unit tangent vector $\mathbf{T}(t)$.

$s'(t) = |\mathbf{v}(t)|$ ____________________ $\mathbf{T}(t) =$ ____________________

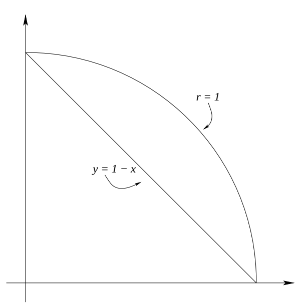
4. (10 points) Suppose that $z = F(u, v, w)$, while $u = f(x, y)$, $v = g(x, y)$ and $w = h(x, y)$. If all these functions are differentiable, what does the chain rule imply about $\dfrac{\partial z}{\partial y}$?

5. (20 points)

(a) Please set up the following double integral as an iterated integral in polar coordinates:

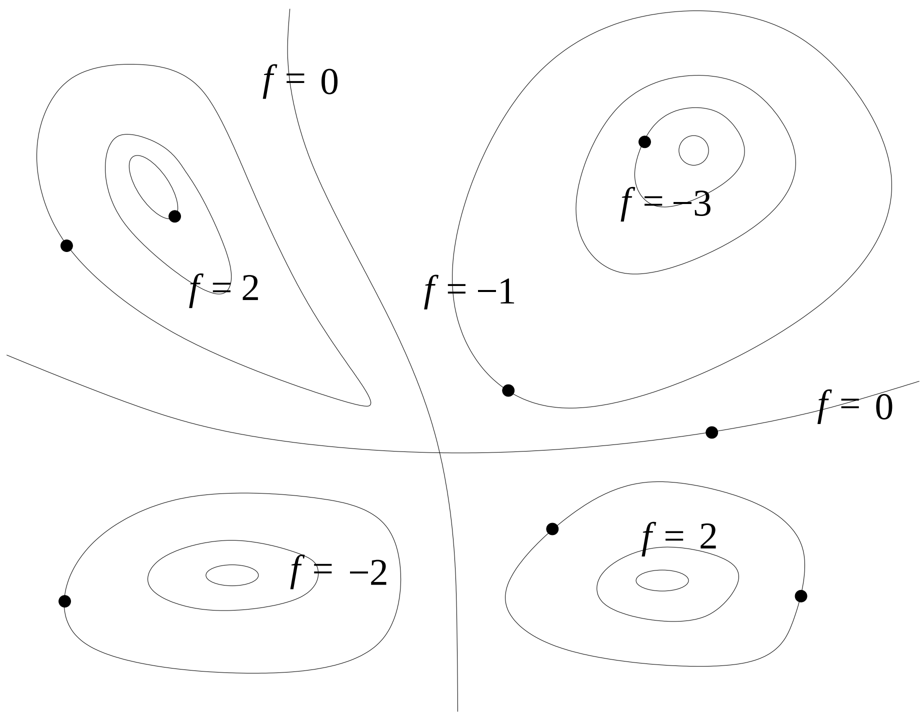
$$\iint_D f(r, \theta)\, dA$$

where D is the region shown in the figure below:



(b) For $f(r, \theta) = 1 + 2 \sin\theta \cos\theta$, please evaluate this integral.

6. (10 points) In the diagram below, the function f takes on the indicated values on each marked curve. Assuming that f is differentiable, please draw a vector arrow at each point • to show ∇f at that point, then draw a short line segment through each point to indicate the direction in which the directional derivative is zero. Next place an asterisk (*) at each point where the gradient is zero.



Finally, please fill in the following blanks:

- The curves in this diagram are called ______________ curves for the function f.
- The one point where the curves cross is a ____________________________.

7. (10 points) Consider a hemispherical dome with radius R sitting on top of the x, y-plane: $x^2+y^2+z^2 = R^2,\ z \geq 0$. Suppose that the dome is filled with a gas whose density decreases linearly with height (so $\delta(z) = \delta_0(1-z/R)$). Please set up **but do not evaluate** an iterated triple integral in spherical or cylindrical coordinates (your choice) which represents the mass of this dome.

Integral: ____

8. (10 points) If **a** and **b** are nonzero vectors, please explain in **one sentence** why $\mathbf{a} \times \mathbf{b} \cdot \mathbf{a} = 0$.

9. (10 points) Consider the function $f(r, \theta) = 2\cos\theta \sin\theta$ for $r \neq 0$, with $f(0, \theta) = 0$. Is this function continuous at the origin (when $r = 0$)? Does $\frac{\partial f}{\partial r}$ exist at the origin? Please explain your answers.