MA1034 Theoretical Calculus IV **Name:** ______________________

Final **B Term, 2017**

Show all work needed to reach your answers.

1. (10 points) For the surface $z = f(x, y) = 3x^2 + xy - 7y - 3y^2 + 2$, please find an equation for the tangent plane at $(1, -1, 8)$.

   Tangent Plane: ______________________

2. (10 points) Consider the vector-valued function $\boldsymbol{f}(x, t) = \langle x\cos(\omega t), x\sin(\omega t)\rangle$ where ω is a constant. Please the Jacobian matrix $J(\boldsymbol{f})(x, t)$

   $$J(\boldsymbol{f})(x, t) =$$

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3. (10 points) Please reverse the order of integration for $\displaystyle\int_1^3 \int_3^{3x} f(x, y)\,dy\,dx$

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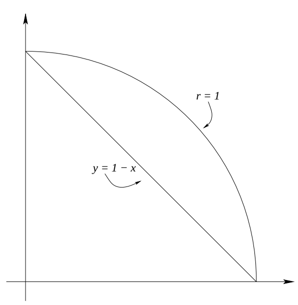
4. (10 points) For the vector function $\boldsymbol{x}(t) = \langle 5t, t^2 + t, e^{-3t} \rangle$ please find the unit tangent vector $\boldsymbol{T}(t)$.

5. (20 points)

   (a) Please set up the following double integral as an iterated integral in polar coordinates:

   $$\iint_D f(r, \theta)\, dA$$

   where D is the region shown in the figure below:



   (b) For $f(r, \theta) = (\sin\theta + \cos\theta)^2$, please evaluate this integral.

6. (10 points) For the surface $z = f(x, y) = x^2 + xy + y^2 + x - y - 10$ please find and classify the critical point.

7. (10 points) Suppose there is a particle whose position on a curve is given by a differentiable vector function. If the arc length for $t \geq 0$ is given by $s(t) = \frac{3}{2}t^2$, and the length of the acceleration vector is $|\boldsymbol{a}(t)| = \sqrt{10}$. Please find the curvature $\kappa(s)$ for this curve.

Curvature $\kappa(s) =$ ________

8. (10 points) Suppose that f and g are continuous real-valued functions defined on some domain $D \subset \mathbb{R}^n$. Please prove that the function $f + g$ is also continuous on D.

9. (10 points) Suppose that $F : \mathbb{R}^3 \to \mathbb{R}$ is a differentiable function, and consider the surface $F(x, y, z) = 0$. Please explain why

$$\frac{\partial z}{\partial y}\frac{\partial y}{\partial x}\frac{\partial x}{\partial z} = -1$$