MA1034 Theoretical Calculus IV **Name:** ____________________

Final **B Term, 2019**

This is a closed book/notes test; no electronic devices allowed. Show all work needed to reach your answers. There are 10 free points on this exam.

1. (10 points) If $u(x, y) = x^2 \cos 7y + 3x^5 \sin 4y$, please find the gradient, ∇u.

$\nabla u(x, y) =$ ____________________

2. (10 points) Suppose that $u = u(x, y, z)$ and that $x = f(s, t) = st^2$, $y = g(s, t) = 3s + 2t$ and $z = h(s, t) = 4t - s$. Please write $\partial u/\partial t$ as explicitly as possible.

$\dfrac{\partial u}{\partial t} =$ ____________________

3. (15 points) Please compute $\iint_D y \, dA$ where D is semicircular disk centered at $(0, 0)$ of radius 3 with $y > 0$.

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4. (20 points) For $t > 0$, consider the vector function $\boldsymbol{x}(t) = \langle t^2/2+3, t^3/3+1\rangle$ which gives the position of a particle moving in the x_1, x_2-plane. Please find the velocity vector $\boldsymbol{v}(t)$, the speed $\dot{s}(t)$, the arc length $s(t)$, and the unit tangent vector $\boldsymbol{T}(t)$. Use $t_0 = 0$, $x_0 = \langle 3, 1\rangle$.

$\boldsymbol{v}(t) =$ ______________________ $\qquad \dot{s}(t) =$ ______________________

$s(t) =$ ______________________ $\qquad \boldsymbol{T}(t) =$ ______________________

5. (10 points) Please compute the following limit, or explain why the limit does not exist:

$$L = \lim_{\substack{(x,\,y)\,\to\,(-2,\,2)\\ y\,\neq\,-x}} \frac{x+y}{x^2-y^2}$$

$L =$ ______________

6. (15 points) Set up but **do not evaluate** an iterated integral equal to

$$\iiint_{\Omega} f(x, y, z)\ dV$$

where Ω is the egg bounded above the x, y-plane by the paraboloid $z = 1 - x^2 - y^2$ and bounded below the x, y-plane by the unit sphere $x^2 + y^2 + z^2 = 1$

7. (10 points) Suppose three planes are randomly arranged in $\mathbb{R}^3$ (so that in particular none of the normal vectors of the three planes are parallel, nor can these normal vectors themselves lie in any single plane). Please describe the intersections of these three planes. Please carefully prove your answer.