MA1034 Theoretical Calculus IV Name Solutions

Quiz 1 B Term, 2018

High: 25
Median: 24
Low: 12

Show all work needed to reach your answers.

1. (5 points) Please find an equation of the plane passing through the origin and containing the vectors $\langle -1, 1, 2\rangle$ and $\langle 1, -1, 0\rangle$.

(+1) $(0,0,0)$ is a point on this plane.

$\vec{N} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 2 \\ 1 & -1 & 0 \end{vmatrix} = \langle 2, 2, 0\rangle$ (+2)

So the plane is $2(x-0) + 2(y-0) + 0(z-0) = 0$ (+1)

$x + y = 0$ (+1) (or any equivalent equation).

#20c §9.3 p.394

2. (8 points) If $\mathbf{u} = \langle 2, 1, 3\rangle$, $\mathbf{v} = \langle 4, -1, 2\rangle$, and $\mathbf{w} = \langle -1, 0, 1\rangle$, please compute $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$.

$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 4 & -1 & 2 \end{vmatrix} = \langle 5, 8, -6\rangle$ (+2) (+2)

$\vec{u} \times \vec{v} \cdot \vec{w} = \langle 5, 8, -6\rangle \cdot \langle -1, 0, 1\rangle$ (+2)

$= -5 + 0 - 6 = -11$ (+1) (+1)

or

$\vec{u} \times \vec{v} \cdot \vec{w} = \begin{vmatrix} 2 & 1 & 3 \\ 4 & -1 & 2 \\ -1 & 0 & 1 \end{vmatrix}$ (+3)

$= 2\begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} - 1\begin{vmatrix} 4 & 2 \\ -1 & 1 \end{vmatrix} + 3\begin{vmatrix} 4 & -1 \\ -1 & 0 \end{vmatrix}$ (+1) (+1) (+1)

$= -2 - 1(6) + 3(-1) = -11$ (+1) (+1)

Answer is a vector (-3)

$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = -11$

#4 II. p.38

3. (12 Points) If the position of a particle is given by the vector function $\boldsymbol{x}(t) = \langle t^2, t^3\rangle$, please find the unit tangent vector, $\mathbf{T}(t)$, and the particle speed, $s'(t)$.

$\vec{v}(t) = \vec{x}'(t) = \langle 2t, 3t^2\rangle = t\langle 2, 3t\rangle$ (+2) (+2)

$s'(t) = |\vec{v}(t)| = \sqrt{4t^2 + 9t^4} = |t|\sqrt{4 + 9t^2}$ (+3) (+3)

$$\mathbf{T}(t) = \begin{cases} \dfrac{\langle 2, 3t\rangle}{\sqrt{4+9t^2}} & t > 0 \\ -\dfrac{\langle 2, 3t\rangle}{\sqrt{4+9t^2}} & t < 0 \end{cases}$$ (+2)

$\hat{T}(0)$ is not defined.

$s'(t) = |t|\sqrt{4 + 9t^2}$ $\quad \forall\, t \in \mathbb{R}$