MA1034 Theoretical Calculus IV Name *Solutions*

Quiz 2 **B Term, 2018**

Show all work needed to reach your answers.

High: 25
Medien: 24
Low: 15

1. (7 points) For the vector function $\boldsymbol{x}(t) = \langle f(t), g(t), h(t) \rangle$, please give the formula for the arc length $s(t)$ for the curve traced out by $\boldsymbol{x}(t)$, assuming that $t_0 = 0$.

No derivatives (−3)

$$s(t) = \int_0^t \sqrt{(f'(\tau))^2 + (g'(\tau))^2 + (h'(\tau))^2}\, d\tau$$

(+1) (+1) (+1) (+1) (+1) (+1) (+1)

2. (8 points) Suppose that a particle is moving along a smooth curve C whose curvature is $\kappa(s) = 5s$. Suppose that the arc length is given by $s(t) = 3t^2$. Please give the acceleration of the particle $\boldsymbol{a}(t)$ in terms of the unit vectors $\boldsymbol{T}(t)$ and $\boldsymbol{N}(t)$.

Here $s'(t) \overset{(+1)}{=} 6t \qquad s''(t) \overset{(+1)}{=} 6$

Since $\vec{a}(t) \overset{(+4)}{=} s''(t)\hat{T}(t) + \underbrace{(s'(t))^2}_{36t^2}\underbrace{\kappa(s(t))}_{5(3t^2)}\hat{N}(t)$

$$\boldsymbol{a}(t) \overset{(+2)}{=} 6\,\hat{T}(t) + 540t^4\,\hat{N}(t)$$

#6
§9.5
p.405

3. (10 points) Please find the (acute) angle θ between the planes $2x - y = 7$ and $-x + y - 3z = 5$. You may give your answer as the inverse trig function of a number.

Here $\vec{N}_1 = \langle 2, -1, 0\rangle$ (+1) and $\vec{N}_2 = \langle 1, -1, 3\rangle$ (+1). Using the angle representation of the dot product, one finds that $\vec{N}_1 \cdot \vec{N}_2 = |\vec{N}_1||\vec{N}_2|\cos\theta$ (+3)

$\Rightarrow \langle 2,-1,0\rangle \cdot \langle 1,-1,3\rangle \overset{(+1)}{=} 2 + 1 + 0 = 3 \overset{(+1)}{=} \sqrt{5}\sqrt{11}\cos(\theta)$

$\Rightarrow \cos\theta \overset{(+2)}{=} \dfrac{3}{\sqrt{55}}$

$\theta = \cos^{-1}\left(\dfrac{3}{\sqrt{55}}\right)$ (+1)