

Quiz 2

B Term, 2019

Show all work needed to reach your answers.

High: 24
Median: 21
Low: 8

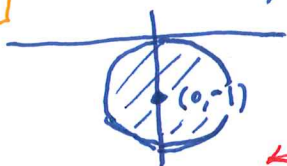
§10.2

1(a)

p.445

1. (4 points) If $f(x, y) = \sqrt{1 - x^2 - (y + 1)^2}$, please specify the domain of f :

For $(x, y) \in \text{Domain}$, we need $1 - x^2 - (y + 1)^2 \geq 0 \Leftrightarrow x^2 + (y + 1)^2 \leq 1$.
So this is all points inside the circle of radius 1 centered at $(0, -1)$.



Domain: $\{(x, y) \in \mathbb{R}^2 \mid x^2 + (y + 1)^2 \leq 1\}$

2. (7 points each) Please compute each limit or explain why the limit does not exist.

III

5e

p.88

(a) $\lim_{\substack{(x, y) \rightarrow (0, 0) \\ y \neq x}} \frac{x^2 - y^2}{x^2 + y^2} = \lim_{r \rightarrow 0} \frac{r^2 \cos^2 \theta - r^2 \sin^2 \theta}{r^2} = \lim_{r \rightarrow 0} [\cos^2 \theta - \sin^2 \theta] = \cos^2 \theta - \sin^2 \theta$
polar coordinates
Depends on the path of approach

Limit:

DNE

III

5d

p.88

(b) $\lim_{\substack{(x, y) \rightarrow (1, 1) \\ y \neq x}} \frac{x^6 - y^6}{x^2 - y^2} = \lim_{\substack{(x, y) \rightarrow (1, 1) \\ y \neq x}} \frac{(x^3 - y^3)(x^3 + y^3)}{(x - y)(x + y)} = \lim_{\substack{(x, y) \rightarrow (1, 1) \\ y \neq x}} \frac{(x - y)(x^2 + xy + y^2)(x + y)(x^2 - xy + y^2)}{(x - y)(x + y)} = \lim_{(x, y) \rightarrow (1, 1)} (x^2 + xy + y^2)(x^2 - xy + y^2) = 3 \cdot 1$

Cancellation

Other cancellations
are possible.

Limit:

3

(c) $\lim_{\substack{(x, y) \rightarrow (0, 0) \\ y \neq 0}} \frac{\cos(xy) - 1}{(xy)} = \left[\lim_{x \rightarrow 0} x \right] \left[\lim_{z \rightarrow 0} \frac{\cos(z) - 1}{z} \right]$
Substitution: $z = xy$
 $= 0 \cdot \left[\lim_{z \rightarrow 0} \frac{-\sin z}{1} \right] = 0 \cdot 0 = 0$
L'Hôpital Rule

Limit:

0