MA1034 Theoretical Calculus IV   Name *Solutions*

Quiz 3   **B Term, 2017**

Show all work needed to reach your answers.

High: 25
Median: 25
Low: 17

1. (5 points) If $f(x, y) = 3x^2y^3 - \sin x$, please find f_x and f_{xy}.

#2
§10.3

$f_x(x, y) =$ $6xy^3 - \cos x$ +3 $f_{xy}(x, y) =$ $18xy^2$ +2

2. (10 points) Please compute each limit or explain why the limit does not exist.

(a) $\lim_{(x,y)\to(0,0)} \frac{x^4 + y^4}{x^2 + y^2} = \lim_{r\to 0} \frac{r^{\cancel{4}\,2}(\cos^4\theta + \sin^4\theta)}{\cancel{r^2}} = \lim_{r\to 0} r^2(\cos^4\theta + \sin^4\theta)$ +2

Use polar coordinates

$= 0$ +3 because $|\cos^4\theta + \sin^4\theta| \le |\cos^2\theta + \sin^2\theta| = 1$

Limit: 0

(b) $\lim_{\substack{(x,y)\to(0,0)\\ y\neq \pm x}} \frac{x-y}{x^2-y^2} = \lim_{\substack{(x,y)\to(0,0)\\ y\neq\pm x}} \frac{\cancel{(x-y)}}{(x+y)\cancel{(x-y)}}$ +2

#3b
§3, p67

But since the denominator goes to zero +3 through both positive and negative numbers, this limit can not exist.

Limit: DNE

3. (10 points)

(a) Can the expression in ~~1~~ 2(b) be defined to be continuous at (0, 0)? Please explain why or why not.

#4b
§3, p.67

Since the limit in 2(b) does not exist, this expression can not be continuous at (0,0)

(b) Can the expression in ~~1~~ 2(b) be defined to be continuous at (1, 1)? Please explain why or why not.

Since $\lim_{\substack{(x,y)\to(1,1)\\ x\neq y}} \frac{x-y}{x^2-y^2} = \lim_{(x,y)\to(1,1)} \frac{\cancel{x-y}}{(x+y)\cancel{(x-y)}} = 1/2$, this expression can be defined continuous at (1,1).