MA1034 Theoretical Calculus IV Name Solutions

Quiz 3 B Term, 2019

Show all work needed to reach your answers.

High: 25
Median: 25
Low: 19

1. (5 points) If $f(x,y) = x^2 + 2xy$, please find f_x and f_{xy}.

III 10a p.90

$f_x(x,y) = 2x + 2y$ (+1 +1 +1) $\qquad f_{xy}(x,y) = 2$ (+2)

2. (10 points) For each $f(x,y)$ defined for $(x,y) \neq (0,0)$, please define $f(0,0)$ to make each function continuous.

(a) $f(x,y) = \dfrac{xy^2 + 3xy + 7x}{x^2 + xy - x} \qquad (x \neq 0)$

$= \dfrac{x(y^2+3y+7)}{x(x+y-1)}$ (+2)

So $\lim_{\substack{(x,y)\to(0,0)\\ x\neq 0}} f(x,y) \overset{+1}{=} \lim_{\substack{(x,y)\to(0,0)\\ x\neq 0}} \dfrac{y^2+3y+7}{x+y-1} \overset{+1}{=} -7$ Hence continuity requires that...

... $f(0,0) = -7$ (+1)

III 6h p.89

(b) $f(x,y) = \dfrac{\sin 4(x^2+y^2)}{x^2+y^2} = 4\left(\dfrac{\sin 4(x^2+y^2)}{4(x^2+y^2)}\right)$ (+2) $= 4\dfrac{\sin u}{u}$

So $\lim_{(x,y)\to(0,0)} f(x,y) \overset{+1}{=} 4 \lim_{(x,y)\to(0,0)} \dfrac{\sin 4(x^2+y^2)}{4(x^2+y^2)} = 4 \underbrace{\lim_{u\to 0} \dfrac{\sin u}{u}}_{1} \overset{+1}{=} 4$

Hence continuity requires that...

... $f(0,0) = 4$ (+1)

3. (10 points)

Please find an equation for the tangent plane to the surface $z = \underbrace{y^2 \cos x}_{f(x,y)}$ at the point on the surface $(\pi/4, 2, 2\sqrt{2})$.

$(x_0, y_0, f(x_0,y_0))$

$z = f(x_0,y_0) + \dfrac{\partial f}{\partial x}(x_0,y_0)(x-x_0) + \dfrac{\partial f}{\partial y}(x_0,y_0)(y-y_0)$ (+1 +1 +1 +1)

$= 2\sqrt{2} + (-y^2\sin x)\big|_{(\pi/4,2)}(x-\pi/4) + (2y\cos x)\big|_{(\pi/4,2)}(y-2)$ (+1 +1 +1 +1)

$z = 2\sqrt{2} - 2\sqrt{2}(x-\pi/4) + 2\sqrt{2}(y-2)$ (+1 +1)

Tangent Plane: $z = -2\sqrt{2}x + 2\sqrt{2}y + \left(\dfrac{\sqrt{2}\pi}{2} - 2\sqrt{2}\right)$ } either