MA1034 Theoretical Calculus IV Name Solutions

Quiz 4 **B Term, 2019**

High: 25
Median: 23
Low: 16

Show all work needed to reach your answers.

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1. (10 points) Please find the unit vector $\boldsymbol{u}$ in the direction in which $f(x,y) = y^2 \sin x$ increases most rapidly at the point $(0,-2)$.

Here $\vec{\nabla} f(0,-2) = \langle y^2 \cos x, 2y \sin x \rangle \Big|_{(0,-2)} = \langle 4, 0 \rangle$ (+1) (+2) (+2)

Since $|\vec{\nabla} f(0,-2)| = 4$, (+2)

(+3)

$\boldsymbol{u} =$ $\langle 1, 0 \rangle$

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2. (10 points) For $x^2(1+\sqrt{y+z^2})e^{xy^2} = 7$, please compute $\partial z/\partial x$ implicitly.

$$\underbrace{0 = \frac{\partial}{\partial x}(7)}_{+1} = \underbrace{\frac{\partial}{\partial x}\left(x^2(1+\sqrt{y+z^2})e^{xy^2}\right)}_{+1} = 2x(1+\sqrt{y+z^2})\cancel{e^{xy^2}} + x^{\cancel{2}}\left(\frac{\cancel{2}z\,\partial z/\partial x}{\cancel{2}\sqrt{y+z^2}}\right)\cancel{e^{xy^2}} + x^{\cancel{2}}(1+\sqrt{y+z^2})\,y^2\cancel{e^{xy^2}} = 0$$

(+2) (+2) (+2)

$\Rightarrow$ Since $e^{xy^2} \neq 0$, if $x \neq 0$,

$$(2+xy^2)(1+\sqrt{y+z^2}) = \frac{-xz\,\partial z/\partial x}{\sqrt{y+z^2}}$$ (+1)

(+1)

$\partial z/\partial x = -\dfrac{(2+xy^2)(1+\sqrt{y+z^2})\sqrt{y+z^2}}{xz}$

provided that $x, z \neq 0$ and that $y+z^2 \geq 0$.

3. (5 points) Suppose that $F : \mathbb{R}^3 \to \mathbb{R}$ is a differentiable function, and that $W = F(x,y,t) \cancel{= 0}$. Moreover, suppose $x = f(t)$ and $y = g(t)$ where f and g are both differentable. According to the chain rule, what is dF/dt?

$$\underbrace{\frac{dF}{dt} = \frac{\partial F}{\partial x}\frac{dx}{dt} + \frac{\partial F}{\partial y}\frac{dy}{dt} + \frac{\partial F}{\partial t}}_{+1}$$ (+1) (+1) (+1) (+1)

F: W → x, y, t
f, g: x, y, t → t