

Quiz 5

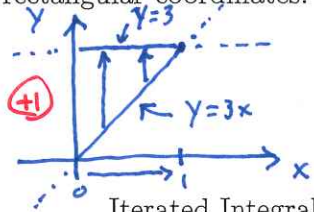
B Term, 2019

Show all work needed to reach your answers.

High: 25
Median: 23
Low: 13

1. (10 points) Please set up an iterated integral (in rectangular or polar coordinates as requested) for the double integral $\iint_D f(x, y) dA$ and each domain D :

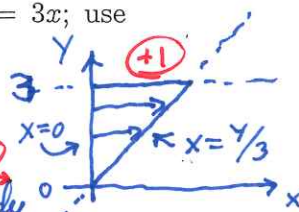
- (a) D is the triangle bounded by the y -axis, the line $y = 3$ and the line $y = 3x$; use rectangular coordinates.



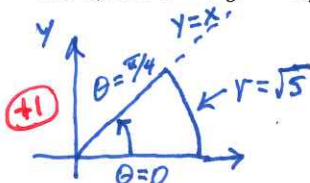
Iterated Integral:

$$\int_0^1 \int_{3x}^3 f(x, y) dy dx$$

$$\text{OR} \int_0^3 \int_0^{y/3} f(x, y) dx dy$$



- (b) D is the disk sector in the first quadrant bounded by the x -axis, the line $y = x$ and the circle $x^2 + y^2 = 5$; use polar coordinates.

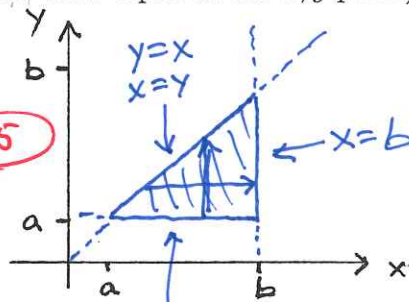


Iterated Integral:

$$\int_0^{\pi/4} \int_0^{\sqrt{5}} f(r \cos \theta, r \sin \theta) r dr d\theta$$

2. (5 points) Consider the Dirichlet formula with $0 < a < b$ so that $[a, b]$ is an interval:

$$\int_a^b \int_a^x f(x, y) dy dx = \int_a^b \int_y^b f(x, y) dx dy$$

What region are these iterated integrals over (please draw a plot in the x, y -plane)?

3. (10 points) Please evaluate the iterated integral

$$\begin{aligned} \int_0^2 \int_{1+x}^{5-x} x^2 dy dx &= \int_0^2 x^2 \int_{1+x}^{5-x} dy dx = \int_0^2 x^2 [(5-x) - (1+x)] dx \\ &= \int_0^2 x^2 (4-2x) dx = \int_0^2 (4x^2 - 2x^3) dx = \left(\frac{4}{3} x^3 - \frac{2}{4} x^4 \right) \Big|_0^2 \\ &= \frac{32}{3} - 8 = \frac{32-24}{3} = \frac{8}{3} \end{aligned}$$

8/3