MA1034 Theoretical Calculus IV Name Solutions

Quiz 2 B Term, 2015

Show all work needed to reach your answers.

1. (10 points) Please find the unit tangent vector and the tangential component of acceleration for $x(t) = \langle \cos t, \sin t, t \rangle$

$\Rightarrow \vec{v}(t) = \vec{x}'(t) = \langle -\sin t, \cos t, 1 \rangle$ (+2) and $s'(t) = |\vec{v}(t)|$

$= \sqrt{(-\sin t)^2 + (\cos t)^2 + 1^2} = \sqrt{2}$ (+2) $\Rightarrow \hat{T}(t) = \vec{v}(t)/s'(t) = \frac{\sqrt{2}}{2}\langle -\sin t, \cos t, 1 \rangle$ (+2)

Secondly $a_T(t) = s''(t) = \frac{d}{dt}(\sqrt{2}) = 0$ (+2) (+2)

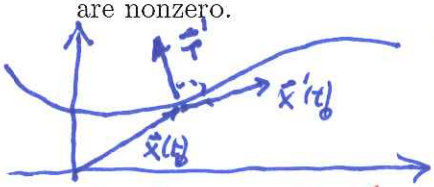
High:	25
Median:	21
Low:	12

unit tangent vector: $\hat{T}(t) = \frac{\sqrt{2}}{2}\langle -\sin t, \cos t, 1 \rangle$

tangential component: $a_{\text{tan}}(t) = 0$

2. (8 points) Suppose that a two-dimensional vector function $\boldsymbol{x}(t)$ smoothly traces out a curve C in the plane. In terms of $\boldsymbol{x}(t)$ and its derivatives, which vector is tangent to C at the point corresponding to $t = t_0$? Which vector is normal (perpendicular) to C at the point corresponding to $t = t_0$? Any tangent or normal vector (expressed in terms of $\boldsymbol{x}(t)$ and its derivatives) will do, and you may assume that all derivatives exist, and all denominators are nonzero.

Since $\hat{T}(t) = \frac{\vec{x}'(t)}{|\vec{x}'(t)|}$, $|\hat{T}(t)| = 1$, $\vec{T}'(t) \perp \hat{T}(t)$.



(-1): Need to evaluate at $t = t_0$

(+4)

tangent vector: $\vec{x}'(t_0)$ or $\frac{\vec{x}'(t_0)}{|\vec{x}'(t_0)|}$

normal vector: $\vec{T}'(t_0) = \frac{d}{dt}\left(\frac{\vec{x}'(t)}{|\vec{x}'(t)|}\right)\Big|_{t=t_0}$ or $\frac{\vec{T}'(t_0)}{|\vec{T}'(t_0)|}$ (+1) (+3)

3. (7 points) Please find the length of $(\ln \cos t, t)$ on the interval $[0, \frac{\pi}{3}]$.

$$L = \int_0^{\pi/3} \sqrt{\left(\frac{-\sin t}{\cos t}\right)^2 + 1^2}\, dt \quad (+2)\ (+2) \quad = \int_0^{\pi/3} \sec t \quad (+1)\, dt$$

$$= \int_0^{\pi/3} \sec t \, \frac{\sec t + \tan t}{\sec t + \tan t}\, dt \quad (+1) = \int_0^{\pi/3} \frac{\frac{d}{dt}(\sec t + \tan t)}{\sec t + \tan t}\, dt$$

$$= \int_1^{2+\sqrt{3}} \frac{du}{u} \quad \text{where } u = \sec t + \tan t$$

$$= \ln(2+\sqrt{3}) \quad (+1)$$