MA1034 Theoretical Calculus IV Name Solutions

Quiz 3 B Term, 2015

Show all work needed to reach your answers.

1. (17 points) Please compute each limit or explain why the limit does not exist.

5 (a) $\lim_{(x,y)\to(0,0)} \dfrac{\sin(4(x^2+y^2))}{x^2+y^2}$

$= \lim_{r\to 0} \dfrac{\sin 4r^2}{r^2}$ (+1)

Let $u = 4r^2$ (+1)

$= \lim_{u\to 0} 4\dfrac{\sin u}{u}$ (+1)

$= 4\lim_{u\to 0} \dfrac{\sin u}{u} = 4$ (+1)

High: 25
Median: 21
Low: 9

Limit: 4 (+1)

6 (b) $\lim_{(x,y)\to(0,0)} \dfrac{x+y}{\sqrt{x^2+y^2}}$

use polar (+2)

$= \lim_{r\to 0} \dfrac{r(\cos\theta+\sin\theta)}{r}$ (+1) $= \cos\theta + \sin\theta$ (+1)

Because this limit would depend on θ (+1)

this limit DNE.

Limit: DNE (+1)

6 (c) $\lim_{\substack{(x,y)\to(-2,3)\\ y\neq -3x/2}} \dfrac{27x^3+8y^3}{9x^2-4y^2}$

$= \lim_{(x,y)\to(-2,3)} \dfrac{(3x+2y)(9x^2-6xy+4y^2)}{(3x+2y)(3x-2y)}$ (+2) Factor (+3)

$= \lim_{(x,y)\to(-2,3)} \dfrac{9x^2-6xy+4y^2}{3x-2y} = \dfrac{108}{-12} = -9$ (+1)

Limit: -9

2. (8 points)

4 (a) Can the expression in 1(a) be defined to be continuous at $(0,0)$? Please explain why or why not.

Yes, because (+2) this limit is 4, the definition can be extended continuously as $\begin{cases} \dfrac{\sin(4(x^2+y^2))}{x^2+y^2} & (x,y)\neq(0,0) \\ 4 \ (+2) & (x,y)=(0,0) \end{cases}$

4 (b) Can the expression in 1(c) be defined to be continuous at $(-2,3)$? Please explain why or why not.

Yes, this expression can be extended continuously in several ways, for example, as $\dfrac{9x^2-6xy+4y^2}{3x-2y}$

$\forall (x,y)\in\mathbb{R}^2$, $2y\neq 3x$