MA1034 Theoretical Calculus IV Name Solutions

Quiz 4 B Term, 2015

Show all work needed to reach your answers.

1. (10 points) Please find an equation of the plane tangent to the surface $z = f(x, y) = x^3 - 3xy + y^3$ at $(-1, 2, 13)$. Check: $f(-1,2) = -1 - 3(-1)(2) + 2^3 = 13$

Tangent Plane: $z = f(-1,2) + \frac{\partial f}{\partial x}(-1,2)(x+1) + \frac{\partial f}{\partial y}(-1,2)(y-2)$ +5

$= 13 - 3(x+1) + 15(y-2)$

$= -3x + 15y - 20$

+2 $\frac{\partial f}{\partial x}(-1,2) = 3x^2 - 3y\Big|_{(-1,2)} = -3$

$\frac{\partial f}{\partial y}(-1,2) = -3x + 3y^2\Big|_{(-1,2)} = 15$ +2

High: 25
Median: 23
Low: 19

Equation: $z = -3x + 15y - 20$ +1

2. (10 points) Suppose $f(x, y, z) = e^x(y + \sin z)$. Please find $\frac{df}{dx}$ for $f(x, x^2, x^3)$.

$\frac{df}{dx} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\frac{dy}{dx} + \frac{\partial f}{\partial z}\frac{dz}{dx}$ +4

$= e^x(y + \sin z) + e^x(2x) + e^x \cos z\,(3x^2)$ +1 +1 +1 +1 +1

$= e^x(2x + y + \sin z + 3x^2 \cos z)$

$\frac{df}{dx} =$ $e^x(2x + x^2 + \sin x^3 + 3x^2 \cos x^3)$ +1

5

3. (5 points) If $z = f(x, y)$, what limit must be zero if $f$ is to be *differentiable* and have *a unique tangent plane* at $(x_0, y_0)$?

$$\lim_{(x,y)\to(x_0,y_0)} \frac{f(x,y) - \left[f(x_0,y_0) + \frac{\partial f}{\partial x}(x_0,y_0)(x-x_0) + \frac{\partial f}{\partial y}(x_0,y_0)(y-y_0)\right]}{d((x,y),(x_0,y_0))}$$