MA143X Theoretical Calculus III Name Solutions

Quiz 1 A Term, 2013

Show all work needed to reach your answers.

(5 points each) As $n \to \infty$, please determine whether each of the following sequences $\{a_n\}$ converges to a limit L or diverges. If it converges, please give the limit, L.

1. $a_n = \dfrac{n^3}{10n^2+1} = \dfrac{n}{10+1/n^2} = \dfrac{1}{\frac{10}{n}+1/n^3}$ (+2)

Form: "$\frac{\infty}{\infty}$" "$\frac{\infty}{1}$" (+2) "$\frac{1}{0}$"

Both of these show that a_n diverges to $+\infty$.

Diverges (+1)

2. $a_n = \dfrac{1+(-1)^n}{\sqrt{n}}$

$0 \le a_n \le \frac{2}{\sqrt{n}}$ (+2) As $n \to \infty$, $\frac{2}{\sqrt{n}} \searrow 0$. (+2)

So $a_n \to 0$ (+1)

$L = 0$

3. $a_n = n\sin(n^{-2}) = \frac{1}{n}\left(\frac{\sin(n^{-2})}{n^{-2}}\right)$ (+2)

As $n \to +\infty$, $\frac{\sin(n^{-2})}{n^{-2}} \to 1$, so $a_n \to 0$ (+2) (+1)

$L = 0$

4. $a_n = \dfrac{\ln((n+1)^3)}{\ln(n^2)} = \dfrac{3\ln(n+1)}{2\ln(n)}$ (+2) (property of logs)

So by l'Hôpital's Rule, (+1)

$\lim_{n\to+\infty} a_n = \frac{3}{2}\lim_{n\to\infty}\frac{n}{n+1} = \frac{3}{2}\lim_{n\to\infty}\frac{1}{1+1/n}$ (+1)

$L = 3/2$ (+1)