

Quiz 2

D. Term, 2013

Show all work needed to reach your answers.

High:	19
Median:	13
Low:	9

1. (4 points) Suppose $S = (1, 3) \cup [4, 7]$. What is $S_{\mathbb{R}}^c$?

$$S_{\mathbb{R}}^c = (-\infty, 1] \cup [3, 4) \cup (7, +\infty)$$

2. (4 points) If X and Y are disjoint sets, what is $X \cap Y$?

$$X \cap Y = \emptyset$$

3. (6 points) Please prove the following: If A and B are sets, then $A \subseteq B$ iff $\mathcal{P}(A) \subseteq \mathcal{P}(B)$

3 ($\Rightarrow$) Suppose $A \subseteq B$. Let $X \in \mathcal{P}(A)$ (i.e., let X be a subset of A). Then $X \subseteq A \subseteq B \Rightarrow X \subseteq B \Rightarrow X \in \mathcal{P}(B)$. Therefore $\mathcal{P}(A) \subseteq \mathcal{P}(B)$. $\blacktriangle$

3 ($\Leftarrow$) Now suppose $\mathcal{P}(A) \subseteq \mathcal{P}(B)$. Let $x \in A$. Then $\{x\} \in \mathcal{P}(A) \subseteq \mathcal{P}(B)$ which implies that $\{x\} \subseteq \mathcal{P}(B) \Rightarrow x \in B$. Therefore $A \subseteq B$. $\blacksquare$

4. (6 points) Please prove the following: If A and B are sets contained in some universe U , then

$$A \cup B^c \subseteq (A^c \cap B)^c$$

($\subseteq$) Suppose $x \in A \cup B^c$. Then $x \in A$ or $x \in B^c$. So $x \notin A^c$ or $x \notin B$. Then $x \notin (A^c \cap B)$. Finally $x \in (A^c \cap B)^c$, which implies that $A \cup B^c \subseteq (A^c \cap B)^c$. QED