MA1971 Name: Solutions

Quiz 5 **D Term, 2013**

Show all work needed to reach your answers.

1. (4 points) Let $a_n \in \mathbb{R}$ and consider the sequence $\{a_n\}$. What does it mean to say that $\{a_n\}$ is *strictly increasing*?

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$$a_n < a_{n+1} \quad \forall\, n \in \mathbb{Z}^+$$

or

$$a_n < a_m \quad \forall\, n < m$$

$\boxed{\text{High: 20} \quad \text{Median: 14} \quad \text{Low: 6}}$

2. Let $f : A \to B$ be a function, and let $S \subseteq B$ and $T \subseteq B$.

(a) (4 points) Please define $f^{-1}(S)$, the *inverse image* of S.

4

$$f^{-1}(S) := \{x \in A \mid f(x) \in S\}$$

(b) (6 points) Please prove that $f^{-1}(S \cap T) \subseteq f^{-1}(S) \cap f^{-1}(T)$.

Let $x \in f^{-1}(S \cap T)$. Then $f(x) \in S \cap T \Rightarrow f(x) \in S$ and $f(x) \in T$. So $x \in f^{-1}(S)$ and $x \in f^{-1}(T) \Rightarrow x \in f^{-1}(S) \cap f^{-1}(T)$. Thus $f^{-1}(S \cap T) \subseteq f^{-1}(S) \cap f^{-1}(T)$.

(c) (6 points) Please prove or disprove: $f^{-1}(S) \cap f^{-1}(T) \subseteq f^{-1}(S \cap T)$

Let $x \in f^{-1}(S) \cap f^{-1}(T)$. Then $x \in f^{-1}(S)$ and $x \in f^{-1}(T)$. So $f(x) \in S$ and $f(x) \in T \Rightarrow f(x) \in S \cap T$. Hence $x \in f^{-1}(S \cap T)$, and thus $f^{-1}(S) \cap f^{-1}(T) \subseteq f^{-1}(S \cap T)$.