MA1971 Bridge to Higher Math **Name:** ____________________________

Final **D Term, 2013**

Show all work needed to reach your answers.

1. (20 points) Consider the implication $A \Rightarrow B$ where A and B are themselves statements. For this implication, please state the following:

   (a) contrapositive: ____________________________

   (b) negation: ____________________________

   (c) direct implication: ____________________________

   (d) converse: ____________________________

2. (20 points) THEOREM: If a sequence $\{a_n\} \subseteq \mathbb{R}$ converges, then the limit is unique.

Proof: Suppose that $a_n \to a$ for some $a \in \mathbb{R}$ and that $a_n \to \alpha$ for some $\alpha \in \mathbb{R}$. To prove that the limit is unique, one must show that ____________. So, given any ____________, there exists ____________ such that ______________ $< \epsilon/2$ and also ______________ $< \epsilon/2$ for all ____________. By the triangle inequality,

$|a - \alpha| \leq$ ____ $< \epsilon$

Since ϵ is arbitrarily small, ____________ and thus the limit is unique.

$\square$

3. (15 points) Let $f : A \to B$ be a function, and let $S \subseteq B$ and $T \subseteq B$. Please prove that $f^{-1}(S \cap T) \subseteq f^{-1}(S) \cap f^{-1}(T)$.

4. (15 points) Please show that

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

Hint: Induction.

5. (15 points) Please negate the following statement about a sequence $\{a_n\}$ and its limit a:

$$\forall N \in \mathbb{Z}^+,\ \exists\ \epsilon > 0\ \text{ s.t. } |a_n - a| < \epsilon\ \forall\ n > N$$

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6. (15 points) Consider the sum of any six consecutive positive integers. What can said about the divisors (factors) of this sum? Please prove your answer.