Exercise Set III

- 1. If p and p + 2 are twin primes and p > 3, prove that 6|(p + 1). By definition, twin primes are primes that differ by exactly 2, for example 17 and 19.
- 2. Show that $\sqrt{3}$ is not a rational number.
- 3. If F_n is the nth Fermat number defined as $F_n := 2^{2^n} + 1$. Prove that $F_n = F_{n-1}^2 2(F_{n-2} 1)^2$. Hint: this statement can be proven with or withour induction.
- 4. Suppose that x and y are both odd positive integers. Please show that $x^2 + y^2$ is not a perfect square. By definition, a *perfect square* is an integer $n = k^2$ for some integer k.
- 5. If $n \in \mathbb{Z}^+$, then 3|n iff three divides the sum of the digits of n.