Exercise Set V

1. Show that the harmonic mean is less than or equal to the geometric mean. Hint: Let $b_k := 1/a_k$ and apply the relationship between the geometric mean and the arithmetic mean to the b_k.

2. Draw all of the possible graphs on four vertices.

3. A *walk* is a alternating sequence of vertices and incident edges of the form

   $$\{v_0, e_1, v_1, e_2, v_2, ..., e_n, v_n\}$$

   where repetition is allowed. A *closed walk* is a walk which begins and ends at the same vertex.

   (a) Please give an example of a closed walk with an even number of edges which contains no cycle.

   (b) Please show that any closed walk with an odd number of edges must contain a cycle.

4. Please show that any planar graph must have a vertex with degree that is less than or equal to 5.