MA1971

Name: Solutions

Quiz 4

High: 20
Median: 16
Low: 3

D Term, 2013

Show all work needed to reach your answers.

1. (10 points) Let $A := \{\ell : \ell \text{ is a line in the Cartesian plane}\}$ and let $x \perp y$ iff x and y are *perpendicular* lines. Please explain why $\perp$ is or is not **reflexive**, **symmetric** and **transitive**.

3 reflexive: No, a line can not be perpendicular to itself $(x \not\perp x)$: $m \neq \frac{-1}{m} \Leftrightarrow m^2 \neq -1$

3 Symmetric: Yes, if $x \perp y$, then $y \perp x$: $m_1 = \frac{-1}{m_2} \Leftrightarrow m_2 = \frac{-1}{m_1}$

3 transitive: No, for lines in the plane, if $x \perp y$ and $y \perp z$, then $x \parallel z$: $m_1 = -\frac{1}{m_2}$ and $m_2 = -\frac{1}{m_3} \Rightarrow m_1 = m_3$

1 General

2. (4 points) What is the key distinction between a **function** and a **relation**?

A relation between sets A and B is any subset of $A \times B$.

A function is a relation which satisfies that $\forall a \in A$, $\exists! b \in B$ such that $f: a \mapsto b$ $((a,b) \in f)$.

3. (6 points) Suppose A and B are sets and $f : A \to B$ is a function. What must be true if f is 1-1? What must be true if f is **onto**?

3 1-1: If $y \in f(A)$ and $y = f(x_1) = f(x_2)$, then $x_1 = x_2 \in A$.

3 onto: $f(A) = B$, i.e. $\forall y \in B$, $\exists x \in A$ s.t. $f(x) = y$.