

Show all work needed to reach your answers.

1. (10 points) Are the nonnegative real numbers, $[0, \infty)$, well-ordered? Please explain why or why not.

$\text{The nonnegative real numbers, } [0, \infty), \text{ is not well-ordered.}$
 $\text{"Well-ordered" requires that every subset of } [0, \infty) \text{ contain a least element.}$ Among many examples, $(0, 1) \subseteq [0, \infty)$ in $(0, 1)$ does not contain its least element, 0.

High: 20
Medium: 17
Low: 3

2. (10 points) Please explain why \mathbb{Q} (the set of rational numbers) is countable.

\mathbb{Q} is countable because it can be put in one-to-one correspondence with \mathbb{Z}^+ . Let $r \in \mathbb{Q}$. Then $r = p/q$ for some $p \in \mathbb{Z}$ and $q \in \mathbb{Z}^+$ where p and q have no common divisors. Now take 0 as the first member of \mathbb{Q} , +1 and -1 as the second and third elements, and the rest of \mathbb{Q} counted based on the value of $|p|+q$ since there are only a finite number of rationals with any given value of $|p|+q$ according to the table →

0	1	2	3	4
	± 1	± 2	± 3	± 4
	$\pm 1/2$	$\pm 2/3$	$\pm 3/4$	$\pm 4/5$
	$\pm 1/3$	$\pm 2/5$	$\pm 3/7$	$\pm 4/9$
	$\pm 1/4$	$\pm 2/7$	$\pm 3/13$	$\pm 4/19$

- Counted \mathbb{Q}^+ rather than \mathbb{Q}
- No explicit count