MA1971 Name: Solutions

Quiz 3 D Term, 2015

Show all work needed to reach your answers.

1. (10 points) Suppose that U is some universal set and that A is a subset of U, that is, $A \subset U$. Using one or more of the axioms, please show that A_U^c (the complement of A in U) exists. Please cite the axiom(s) you use.

High: 20
Median: 20
Low: 8

+4 By Axiom III (Specification), if U and A are sets with $A \subset U$, then +4 $\{x \in U \mid x \notin A\}$ must be a set ($Q(x) =$ "$x \notin A$" is the predicate). By definition, +2 this set is A_U^c.

2. (10 points) Please show that $\underbrace{1 + 3 + 5 + \ldots + (2n-1)}_{\sum_{k=1}^{n}(2k-1)} = n^2$.

This can be proven by induction. +1 First notice that $\sum_{k=1}^{1}(2k-1) = 1 = 1^2$, so the result is true for $n=1$. +4

Now suppose that the result holds for n, and show that the result holds for $n+1$: +1

$$\underbrace{1+3+5+\ldots+(2n-1)}_{n^2} + (2(n+1)-1) = n^2 + (2(n+1)-1)$$
+1 +1
$$= n^2 + 2n + 1$$
+1
$$= (n+1)^2$$

So by induction, 1 $\sum_{k=1}^{n}(2k-1) = n^2$.