

Quiz 4

D Term, 2015

Show all work needed to reach your answers.

1. (10 points) Please complete the following multiplication table for $\mathbb{Z}_{11}$ (multiplication mod 11), then complete the list of equivalence classes of the form $\{x, -x, \bar{x}, -\bar{x}\}$

5

$\times$	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	<u>1</u>	2	3	4	<u>5</u>	6	7	8	9	10
2	0	2	4	<u>6</u>	8	10	1	<u>3</u>	5	7	9
3	0	3	<u>6</u>	9	<u>1</u>	4	7	10	2	<u>5</u>	8
4	0	4	<u>8</u>	1	<u>5</u>	9	2	6	10	<u>3</u>	7
5	0	5	10	<u>4</u>	<u>9</u>	3	8	2	7	<u>1</u>	6
6	0	6	1	<u>7</u>	<u>2</u>	8	3	9	4	<u>10</u>	5
7	0	7	3	10	<u>6</u>	2	<u>9</u>	5	<u>1</u>	8	4
8	0	8	5	2	<u>10</u>	7	<u>4</u>	1	<u>9</u>	6	3
9	0	<u>9</u>	7	5	<u>2</u>	1	<u>10</u>	8	6	4	2
10	0	10	9	<u>8</u>	<u>7</u>	6	5	4	3	<u>2</u>	<u>1</u>

High: 20

Median: 20

Low: 14

5 $\mathbb{Z}_{11} = \{0\} \cup \{1, \underline{10}\} \cup \{2, \underline{9}, \underline{6}, \underline{5}\} \cup \{3, \underline{8}, \underline{4}, \underline{7}\}$

2. (10 points) If $x, y \in \mathbb{Z}^+$ with x even and y odd, please show that $x^2 + y^2 = 1 \pmod{4}$.

Since x is even, ⁽⁺³⁾ $x = 2k$, $\forall k \in \mathbb{Z}^+$. Since y is odd, ⁽⁺³⁾ $y = 2j - 1$, $\forall j \in \mathbb{Z}^+$. Then $x^2 + y^2 =$
 $(2k)^2 + (2j - 1)^2 \stackrel{(+2)}{=} 4k^2 + 4j^2 - 4j + 1 \stackrel{(+1)}{=} 4(k^2 + j^2 - j) + 1$
 $\stackrel{(+1)}{=} 1 \pmod{4}.$

QED