MA1971 Bridge to Higher Math Name:

Final

D Term, 2015

Show all work needed to reach your answers.

- 1. (18 points) Several short-answer questions:
 - (a) What do each of the following symbols mean? (Please write meaning after symbol.) $\exists \qquad \forall \qquad \exists!$
 - (b) What is the meaning of \aleph_0 (aleph naught)?
 - (c) If $x \in \mathcal{P}(A)$ what is the relationship between x and A? ($\mathcal{P}(A)$ is the power set of A.)
- 2. (17 points) Suppose that U is some universal set, and suppose that $A, B \subset U$. Please prove that $A^c \cap B \subset (A \cup B^c)^c$.

- 3. (20 points) Which of the following sequences (a) **must converge**, (b) which **must diverge**, and (c) which **might** converge or **might** diverge?
 - (a) A decreasing sequence of positive real numbers.
 - (b) An oscillating sequence of real numbers whose oscillation amplitude decreases to 1.
 - (c) An increasing sequence of real numbers.
 - (d) A decreasing sequence of rational numbers whose greatest lower bound is a negative real number.
 - (e) A sequence of real numbers which is bounded both above and below.
- 4. (15 points) Suppose that the sum of the digits of a number $n \in \mathbb{Z}$ is divisible by 9. Please show that 9|n. Hint: If n has N + 1 digits, let

$$n = a_N a_{N-1} \dots a_2 a_1 a_0 = \sum_{k=0}^N a_k 10^k$$

5. (15 points) Please give the contrapositive of the following statement:

If x > 0 but y < 0, then z = 0 or $q \in \mathbb{Q}$.

6. (15 points) A graph which is the union of disjoint trees is sometimes called a forest. Suppose that $T := \{T_1, T_2, ..., T_k, ...\}$ be a set of trees, and suppose that

$$F := \bigcup_{k=1}^{|T|} T_k$$

is such a forest (here |T| be the number of trees in the forest). Let $|V_k|$ and $|E_k|$ be the numbers of vertices and edges in T_k , the k-th tree; let |V| and |E| be the total numbers of vertices and edges overall in the forest. Please find a formula relating |T|, |V| and |E|. Also starting from the Euler formula, please prove that your formula is correct.