## MA1971 Bridge to Higher Math Name:

## Final

D Term, 2021

Show all work needed to reach your answers.

- 1. (10 points) If  $A = \{2, 4, 8\}$ , then the power set of A is
- 2. (20 points) Consider the implication  $A \Rightarrow B$  where A and B are themselves statements or predicates. For this implication, please state the following:
  - (a) contrapositive:
    (b) converse:
    (c) negation:
    (d) inverse:
- 3. (10 points) Please give (a) the contrapositive and then (b) the negation of the following statement: "If xy is an irrational number, then y > 6 but x < 0." Please avoid the use of the words "not" and "no".

(a) Contrapositive:

(b) Negation:

4. (20 points) Please show that  $\sqrt{5}$  is irrational.

5. (10 points) Consider a sequence  $\{a_n\}$  where  $a_n = p_n/q_n$  and  $0 < p_n < q_n$  (so each element of the sequence is a fraction). Suppose that  $a_n$  is increasing. Does  $\{a_n\}$  necessarily converge? Please either explain why it converges, or give a counterexample to show that such a sequence might diverge.

6. (20 points) Please explain why  $i^2 + j^2$  is never equal to 3 (mod 4), that is,  $i^2 + j^2 \neq 4k + 3$  for any  $i, j, k \in \mathbb{Z}$ .

Hint: Consider the cases where i and j are each either even or odd; what do these imply?

7. (10 points) Consider the graph below; it is one possible drawing of  $K_5$ , the complete graph on five vertices. Recall that by the Euler formula, one might expect that |V| - |E| + |F| = 2. But for this graph, it seems that |V| = 5, |E| = 10 and |F| = 8, meaning that the Euler formula is not satisfied. Please explain what is wrong here.

