Bridge to Higher Mathematics, MA 1971 D21

Exercise Set II

1. Let A and B be subsets of a universe U. Please prove the second De Morgan's law:
$$(A \cap B)^{c} = A^{c} \cup B^{c}$$
2. Prove that if A, B and C are sets, and if $A \subset B$ and $B \subset C$, then $A \subset C$.
3. If $U := [0, 10]$, $A := [3, 7)$ and $B := \{3, 6, 9\}$, then what are A^{c}_{U}, $A^{c}_{\mathbb{R}}$ and B^{c}_{U} ?
4. Let A and B be sets. Please prove or disprove:
$$\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$$
Hint: Counterexample
5. Prove that for each $n \in \mathbb{Z}^{+}$,
    1. $$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$
    2. $$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$
6. Please find two distinct proofs that for any $n \in \mathbb{Z}^{+}$, then 6 divides $n^3 - n$, that is, $6|(n^3 - n)$.
7. Suppose A and B are sets with $A \subset B$. Given the standard definition of A^{c}_{B}, use the axioms to show that this complement exists.
8. In terms of axiomatic set theory, please explain why a "set" containing all sets is not a set.
9. Is $\emptyset$ the same as $\{\emptyset\}$? Explain why or why not. Hint: Cardinality.
10. Please construct on the basis of the axioms a set containing exactly three elements.