Exercise Set III

1. If p and $p+2$ are twin primes and $p > 3$, prove that $6|(p+1)$. By definition, *twin primes* are primes that differ by exactly 2, for example 17 and 19.
2. Show that $\sqrt{3}$ is not a rational number.
3. If F_n is the n^{th} Fermat number defined as $F_n := 2^{2^n} + 1$. Prove that $F_n = F_{n-1}^2 - 2(F_{n-2} - 1)^2$. Hint: this statement can be proven with or without induction.
4. Suppose that x and y are both odd positive integers. Please show that $x^2 + y^2$ is not a perfect square. By definition, a *perfect square* is an integer $n = k^2$ for some integer k.
5. If $n \in \mathbb{Z}^+$, then $3|n$ iff three divides the sum of the digits of n.