Exercise Set IV

1. A real number x is called a *root number* iff $x = \sqrt[n]{m}$ (that is, x is the n-th root of m) where n and m are positive integers. Please show that there are a countable number of root numbers.

2. Please define the continued fraction

$$\cfrac{1}{3+\cfrac{1}{3-\cfrac{1}{3-\cfrac{1}{3-\ddots}}}}$$

   as a sequence, then show that this sequence converges, and find its value.

3. Please define the continued radical

$$\sqrt{x+\sqrt{x+\sqrt{x+\sqrt{x+\ldots}}}}$$

   as a sequence of functions (expressions), then show that this sequence converges, and find its value as a function of x (expression in x).

4. Please define the continued fraction

$$\cfrac{1}{2+\cfrac{1}{2-\cfrac{1}{2+\cfrac{1}{2-\ddots}}}},$$

   as a sequence, then show that the odd elements of this sequence oscillate with decreasing amplitude of oscillations, and thus converge to some real number.