

### Exercise Set V

1. Show that the harmonic mean is less than or equal to the geometric mean. Hint: Let  $b_k := 1/a_k$  and apply the relationship between the geometric mean and the arithmetic mean to the  $b_k$ .
2. Draw all of the possible graphs on four vertices. Which ones are connected and which are disconnected?
3. A *walk* is a alternating sequence of vertices and incident edges of the form

$$\{v_0, e_1, v_1, e_2, v_2, \dots, e_n, v_n\}$$

where repetition is allowed. A *closed walk* is a walk which begins and ends at the same vertex.

- (a) Please give an example of a closed walk with an even number of edges which contains no cycle.
  - (b) Please show that any closed walk with an odd number of edges must contain a cycle.
4. Please show that any planar graph must have a vertex with degree that is less than or equal to 5.